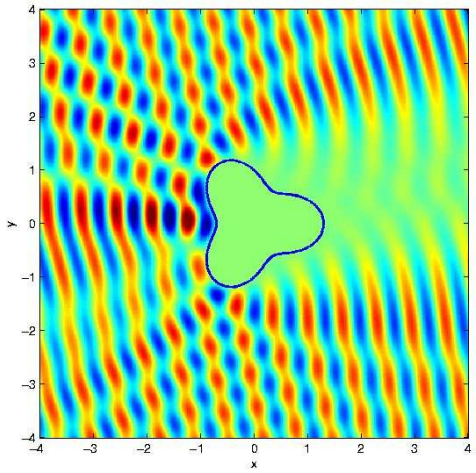
Math 116: Applied Mathematics

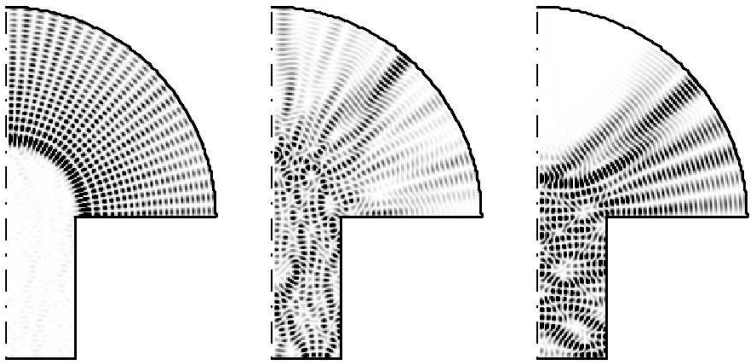
Numerical methods for PDEs and waves

Alex Barnett, Fall 2008, Tu & Th 10:00-11:50am (10A)



2D wave scattering from an obstacle

Questions / input? ahb@math.dartmouth.edu



mushroom eigenmodes: integrability vs quantum chaos

The Laplace equation (describing steady-state diffusion, heat flow, electrostatics) and Helmholtz equation (linear waves, acoustics, electromagnetics, optics, quantum) are linear PDE boundary value problems, ubiquitous in modeling the real world. They may be solved numerically by recasting the problem onto the boundary; this is more efficient at short wavelengths (and easier to code) than standard discretization methods. You will build codes, analyse their errors, explore phenomena in wave scattering and quantum chaos (short-wavelength asymptotics), connect with current research. Along the way you'll pick up some general applied math and numerical analysis.

Hands-on numerical exploration, current mathematics, and beautiful pictures!

Tentative outline: Overview of PDEs and solution methods; numerical integration and quadrature; Method of Particular Solutions for interior BVPs, expansions at singular corners; eigenvalue problems, bounding eigenvalues, isospectral drums, quantum chaos; potential theory, Fredholm theory for compact operators, boundary integral methods, Nyström method, exterior wave scattering.

Optional topics/projects: Vekua's theory of analytic PDEs, dielectrics, periodic lattices, Fast Multipole Method, eigenmodes of manifolds of constant curvature...

- Homeworks: weekly, mixture of computer investigations (in a language of your choice, *e.g.* Matlab, C, C++, Maple) and pencil-and-paper theory.
- Project: during the final few weeks you will choose then work on a topic (numerical or analytical), give a class presentation and write up. *See website for past projects*

Prerequisites: Math 22 (lin alg), 23 (diff eq). Very helpful: 43 (complex), 46 (applied math), 35/63 (real analysis). Some computer programming highly recommended. Undergraduates welcome by permission of instructor.