MATH 108 – TOPICS IN COMBINATORICS: COMBINATORIAL REPRESENTATION THEORY

PROF. ROSA ORELLANA

Prerequisites: Linear algebra and algebra (M31, M71 or M101). No prior knowledge of combinatorics or representation theory is expected.

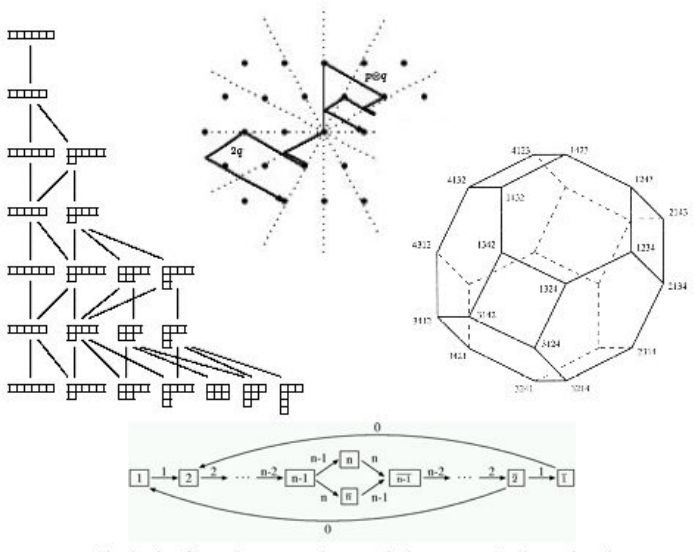
Description: This course is an introduction to algebraic combinatorics. Symmetric function theory and its connections to representation theory and algebraic geometry have dominated the world of algebraic combinatorics over the past twenty-five years or so. Symmetric function theory has a long linked tableaux combinatorics to representation theory of the symmetric group and the general linear group and more recently connections have developed to the representation theory of other groups. On the other hand, tableaux combinatorics and symmetric functions are a useful tool to study enumerative questions about permutations, partitions, and topics such as Polya enumeration theory. The goal of this course is to introduce the student to the following topics:

(1) An overview of representations and characters of finite groups.
(2) A more detailed study of the representation theory of the symmetric group, including Young symmetrizers, specht modules, branching rules, and Gelfand-Tzetling bases.
(3) The Vershik-Okounkov approach to the representation theory of the Symmetric group.
(4) Some representation theory of the general linear group.
(5) Symmetric functions, including Schur functions.
(6) Schur-Weyl duality.
(7) Young tableaux, the hook-length formula, and the RSK algorithm.

Recommended books (no required text - class notes will be available):

- William Fulton and Joe Harris: Representation theory a first course.
- Bruce Sagan: The Symmetric group.
- William Fulton: Young Tableaux.
- Richard Stanley: Enumerative Combinatorics 2.
- Ian Macdonald: Symmetric Functions.

Grading: There will be homework about every other week and class presentations. There will be no formal exams. Students that have already passed all qualifying exams will be excused from the homework assignments.



Clockwise from the top: path crystal elements, a 3-dimensional permutohedron, a tableau crystal, and the partition algebra tower