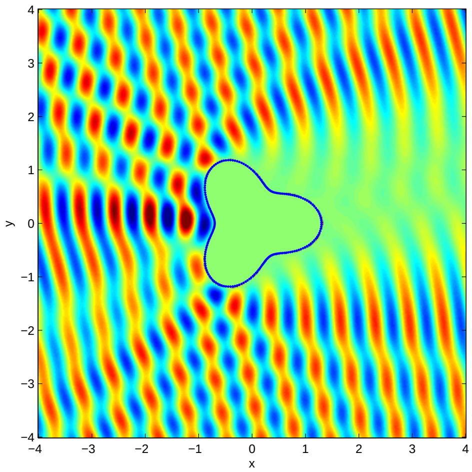
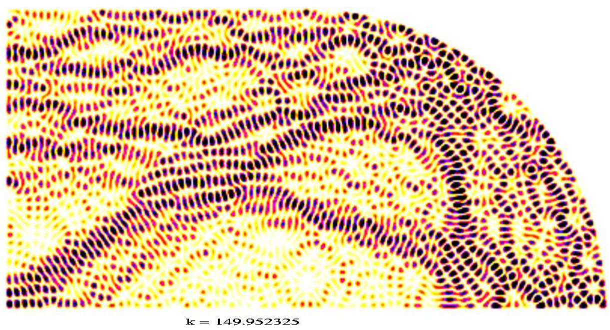
Math 116: Boundary methods and wave asymptotics

Alex Barnett, Winter 2006, Tu & Th 2pm-3:50pm (2A)



2D wave scattering from an obstacle

Questions / input? `ahb@math.dartmouth.edu`



stadium eigenmode: quantum chaos!

My goal is for you to learn the theory and practice of *boundary-based* numerical methods for partial differential equations. I expect to focus on Helmholtz's equation, which describes linear waves (acoustics, electromagnetics, optics, etc) in the frequency domain, and is incredibly useful in engineering/physics. Boundary methods are easier to code than Finite Elements, and are very efficient especially at short wavelengths. Therefore you can use tools you build to explore some wave asymptotics such as WKB, ray methods, and *quantum chaos*, or spectral geometry.

Hands-on numerical exploration, current mathematics, and beautiful pictures!

Tentative outline: Potential theory, Fredholm theory for compact integral operators, Boundary Integral Equations in 2D, interior boundary-value problems *e.g.* Laplace's equation, exterior wave scattering, convergence of numerical errors. Method of Particular Solutions, basis approximation, eigenvalue problems. Short-wavelength asympotics: ray optics, integrable vs chaos, quantum chaos. *Optional topics/projects:* handling corners, dielectric resonators, 3D, Maxwell, Fast Multipole Method, inverse scattering, mode statistics, scars, isospectral drums, eigenmodes in negatively-curved space...

Assessment: (Note you may work in groups of 2-3)

- Homeworks fortnightly (3-4 total): chiefly computer coding & investigations (in a language of your choice, *e.g.* Matlab, C, C++, Maple), some analysis.
- Project: during the final few weeks you will choose then work on a topic (numerical or analytical), give a class presentation and write up.

Prerequisites: 22 (linear algebra), 23 (differential equations). Helpful: 63 (real analysis). Some computer programming recommended (otherwise be willing to dive in and get your feet wet!) Undergraduates welcome by permission of instructor.