Math 104: Differential Topology

ORC syllabus

This course is an introduction to differential topology. Topics include smooth manifolds, tangent and cotangent bundles, vector fields, Lie groups, differential forms, and integration, with additional topics covered at the instructor's discretion.

References.

[BT] Raoul Bott and Loring W. Tu, *Differential forms in algebraic topology*, Springer, 1982.

[GP] Victor Guillemin and Alan Pollack, *Differential topology*, AMS Chelsea, 2010.

[L] John M. Lee, *Introduction to smooth manifolds*, 2nd ed. (hardcover), Springer, 2006. [Note: there is a huge difference in chapter content between the soft and hard cover versions.]

[S] Michael Spivak, *A comprehensive introduction to differential geometry, Vol. 1*, 2nd ed., Publish or Perish, 1979.

[W] Frank W. Warner, *Foundations of differentiable manifolds and Lie groups*, Springer, 1983.

Core differential topology.

1. Smooth manifolds (and manifolds with boundary). Charts, smooth maps, partitions of unity. [S, 1–2; W, 1; GP, 1–2; L, 1–2]

2. Tangent vectors, tangent spaces. Differential of a smooth map. Computations in coordinates. [S, 3; W, 1; GP, 1; L, 2]

3. Tangent bundles, smooth vector fields. Velocity vectors of curves, alternative definitions of the tangent space. Vector bundles, sections, operations. [S, 3; W, 1; GP, 1; L, 3; L, 10]

4. Inverse function theorem. Implicit function theorem. Rank theorem. [S, 2; W, 1; GP, 1; L, Appendix C, 2]

5. Submersions, immersions, embeddings. Immersed and embedded submanifolds. Graphs of smooth functions, level sets, critical points. Tangent spaces to submanifolds. [S, 2; W, 1; GP, 1; L, 4–5]

6. Measure zero sets. Sard's theorem. Whitney embedding theorem. [S, 2; W, 1; GP, 1; L, 6]

7. The cotangent bundle. Pullbacks. [S, 4; W, 1; L, 11]

8. Tensors. Differential of a function, differential forms. [S, 4, 7–8; BT, 1; W, 2; GP, 4; L, 11–12, 14]

9. Integration on Manifolds, Stokes's Theorem. [S, 4, 7–8; BT, 1; W, 2; GP, 4; L, 15–16]

10. De Rham cohomology. [S, 8, 11; BT, 1; W, 4; GP, 4]

Additional topics that may vary from year to year.

1. Mayer-Vietoris theorem. [S 8, 11; BT, 1; W, 4; GP, 4]

2. Distributions. Integral submanifolds. [S 6; W, 1]

3. Lie groups and homogeneous spaces. [S, 10; W, 3; L, 7]

4. Cohomology and the pairing between the bordism group and cohomology by Stokes's theorem. [L, 16]

5. Riemannian metrics. [L, 13]