Math 106: Stochastic processes and uncertainty quantification

ORC syllabus

Stochastic modeling and uncertainty quatification are central to the study of many problems in physics, engineering, finance, evolutionary biology and medicine. This course introduces theoretical concepts in probability theory and key methods for stochastic processes and uncertainty quantification.

The topics of this course will alternate between odd and even years. In even years, topics will include basic concepts of probability, generat- ing functions, Markov chains, random walks, Markov and Non-Markov processes, and diffusion theory. Applications to the natural sciences will be made. In odd years, the course will focus on data-driven meth- ods, with applications in data science, machine learning, and numerical weather prediction. Topics will include statistical inference, random sampling, stochastic processes, polynomial chaos, and data assimila- tion. The course will also introduce standard computation libraries in MATLAB and Python.

Even years:

References.

1. Karlin, S., & Taylor, H. E.. *A first course in stochastic processes*, Academic Press, 1975.

2. Bailey, N. T., *The elements of stochastic processes; with applications to the natural sciences*, John Wiley & Sons Inc. 1964.

3. Ewens, W. J., *Mathematical population genetics 1: theoretical introduction* Springer Science & Business Media, volume 27, 2012.

Stochastic processes with applications

1. Basic concepts of probability and generating function approach

2. Applications of generating function approach

3. Markov chains

4. Random walks

5. Continuous time Markov processes

6. Diffusion theory

7. Birth and death processes

8. Branching processes

Odd years:

References.

1. K. Law, A. Stuart, K. Zygalakis, *Data Assimilation*.

2. C. Robert, G. Casella, *Monte Carlo Statistical Methods*.

Stochastic processes and uncertainty quantification.

1. **Introduction:** Uncertainty and data-driven quantification, review of probability, information theory.

2. **Statistical Inference:** Parametric inference, nonparametric inference, Bayesian inference.

3. **Random Sampling:** Monte Carlo, importance sampling, Markov Chain Monte Carlo.

4. **Special Topics:** Hilbert space, smoothing using orthogonal functions.

5. **Stochastic Processes:** Brownian motion, stochastic differential equations, stationary stochastic processes.

6. **Polynomial Chaos:** Karhunen-Loeve expansion, generalized polynomial chaos.

7. **Data Assimilation:** Kalman filter, approximate Gaussian filters.

8. **Advanced Data Assimilation:** Ensemble square root filter, particle filter, localization and inflation.

9. **Challenges of High-dimensional Spaces:** Sampling in high-dimensional spaces, data assimilation in high-dimensional spaces.