# Algebra preliminary exam syllabus

**References.**

[DF] David Dummit and Richard Foote, *Abstract algebra*, 3rd ed., John Wiley and Sons, 2004.

[FIS] Stephen H. Friedberg, Arnold J. Insel, and Lawrence E. Spence, *Linear algebra*, 4th ed., Pearson, 2002.

**Linear algebra.**

1. Fundamentals of linear algebra, including dual spaces. [FIS, 1.1–4.4]

2. Eigenvalues. Eigenvectors and eigenspaces. Characteristic polynomial. Diagonalization. Cayley–Hamilton theorem. [FIS, 5.1–5.2, 5.4]

3. Inner product spaces. Norms. Orthogonal basis. Gram–Schmidt orthogonalization process. Orthogonal complements. [FIS 6.1–6.2]

4. Adjoint. Normal and self-adjoint operators. Unitary operators. Orthogonal transformations. Spectral theorem. Bilinear and quadratic forms. [FIS 6.3–6.6, 6.8]

5. Jordan canonical form. Minimal polynomial. Rational canonical form. [FIS 7.1–7.4]

**Abstract algebra.**

1. Fundamentals of group theory. [DF 1.1–3.5]

2. Group actions. Permutation representation. Cayley's theorem. The class equation. Automorphism groups. Sylow's theorem. [DF 4.1–4.5]

3. Direct products. Structure of finitely generated abelian groups. Semidirect products. Classifying groups of small order. [DF 5.1–5.5]

4. Fundamentals of ring theory. [DF 7.1–7.6]

5. Euclidean domains, PIDs, UFDs. [DF 8.1–8.3]

6. Polynomial rings. Irreducibility criteria. [DF 9.1–9.5]