Network Analysis of Opinion Formation on Access to Health Care

Lillian Eisner and Casey Vaughan
Advisor: Nishant Malik | MATH 46: Introduction to Applied Mathematics

Abstract

Although infant mortality rates are falling in the United States, the U.S. still has higher infant mortality than comparable wealthy nations [1]. This study examined opinion formation regarding reproductive care for women to understand its effect on vulnerability to infant mortality. Using recent public opinion and health data, we found a compelling relationship between public opinion of reproductive health and infant mortality. Utilizing a Holmes and Newman network model, we used differential and bifurcation analysis to analyze changes in opinion and the rate at which opinion formation occurs in a social network. We concluded that the rate at which opinions change impacts opinion formation within a network and thus the vulnerability to infant mortality.

Motivation

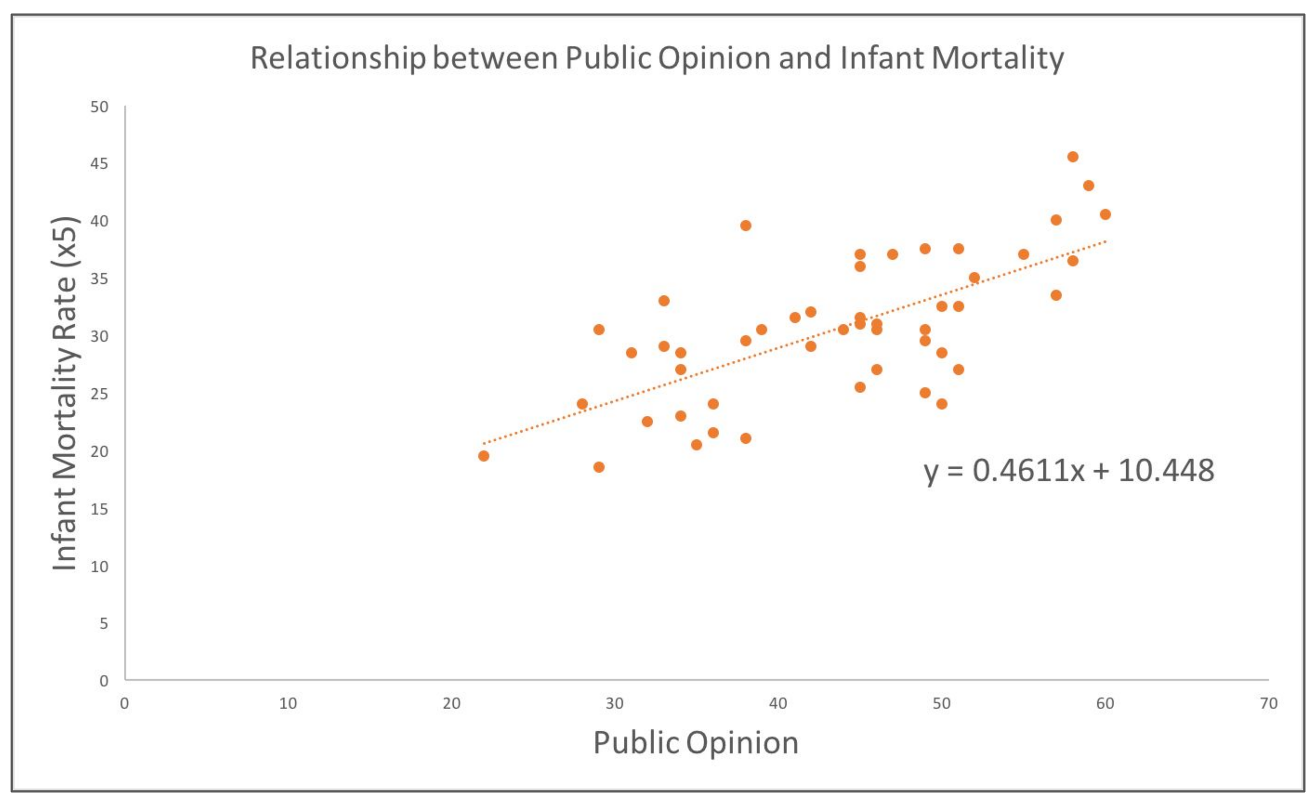


Figure 1: Public opinion (x-axis) vs. Infant Mortality Rates (y-axis)
Vulnerability Equation: Vulnerability = (0.4611(percent anti-choice) + 10.448)/5

Utilizing data obtained from a CDC study that spanned from 2005 to 2016, we noticed that the rates of infant mortality varied greatly depending on the state and were higher than rates in peer countries [2]. We suspected that this may have been a result of the access to health care within communities across each state. To analyze this trend, we decided to look at the percentages of different opinions regarding the legality of abortion in each state because of the relationship between abortions, health care, and infant mortality found in a Religious Landscape study [3]. Plotting public opinion against infant mortality in the last two years, we saw there was a linear relationship between the two and used this relationship as the foundation of our vulnerability model.

Assumptions

- N and K are constant values within the system
- No heterophily exists in the model (no connection formation with opposite node; Holmes-Newman model)

Dynamical System

We utilized the Holme and Newman model for opinion formation in networks when constructing our system of differential equations [4]. In this system [AA] = number of connections in the system between two nodes with opinion A, [BB] = number of connections in the system between two nodes with opinion B, and [AB] = number of connections in the system between a node with opinion A and a node with opinion B. Substituting *u* = ([AA] + [BB])/K, *v* = ([AA] - [BB])/K, and *w* = ([A] - [B])/N, with *N* number of nodes, *K* number of connections (edges), and ϕ as the probability of changing opinion, we simplified the equations to be:

$$\frac{du}{dt} = \frac{N}{K}\frac{1-u}{1-v^2}[1 - vw + \gamma(1 - 2u + v^2)],$$

$$\frac{dv}{dt} = \frac{N}{K}(1-2\phi)\frac{1-u}{1-v^2}(v-w),$$

$$\frac{dw}{dt} = 2(1-\phi)\frac{1-u}{1-v^2}(v-w),$$

Bounds:
$0 \le u \le 1$
$-1 \le v \le 1$
$-1 \le w \le 1$

Parameters:
[A] = # nodes with opinion A
[B] = # nodes with opinion B
N = [A] + [B]
K = [AA] + [AB] + [BB]
ϕ = constant manipulated for different network constructions
$\gamma = [2K(1-\phi)]/N$

We were able to find our set of critical points. The first set occurs when the probabilities of a node with opinion A becoming a node with opinion B, and vice versa, are equivalent. The final point occurs when there are no [AB] connections in the system, and [AB] = 0.

(a) $u = \frac{1}{2}\left(1 - \frac{1}{\gamma}\right)v^2 + \frac{1}{2}\left(1 + \frac{1}{\gamma}\right), \quad v = w,$

(b) $u = 1,$

This system and critical points can be represented three dimensionally.

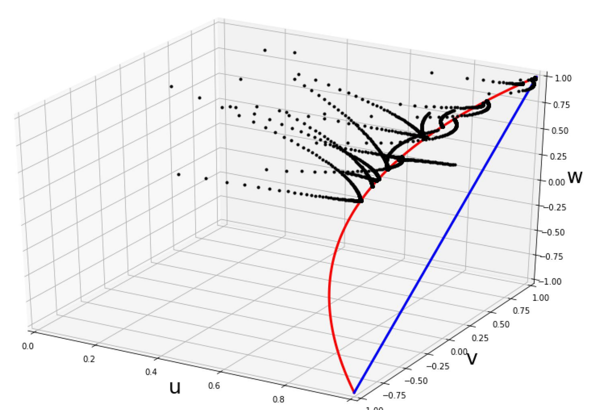


Figure 2: visualization of differential equation system du, dv, dw with ϕ = 0.5 ($\gamma > 1$)

The red line is a dimensional attractor and a set of critical points where:

$u = \frac{1}{2}\left(1 - \frac{1}{\gamma}\right)v^2 + \frac{1}{2}\left(1 + \frac{1}{\gamma}\right), \quad v = w,$

The black dotted lines are the trajectories from various random initial states

The blue line is the unstable critical point where *u* = 1 and *v* = *w*

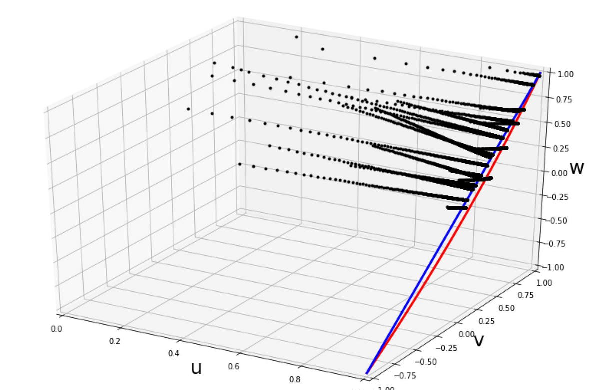


Figure 3: visualization of differential equation system du, dv, dw with ϕ = 0.8 ($\gamma < 1$)

Dimensional attractor equation plotting (red line):

```
yf=np.linspace(-1, 1, 100)
xf=1.0/2.0*(1.0-1.0/gamma)*yf**2+1.0/2.0*(1.0+1.0/gamma)
zf=yf
```

Bifurcation

Eigenvalues corresponding to critical points (a) and (b):

(a)
$$\lambda = \begin{cases} 0, \\ -\frac{N}{K}(\gamma - 1) \lesseqgtr 0, \\ \left(1 - \frac{1}{\gamma}\right)\left(\frac{N}{2K}(1-2\phi) - (1-\phi)\right) \lesseqgtr 0 \quad (\gamma \gtrless 1), \end{cases}$$

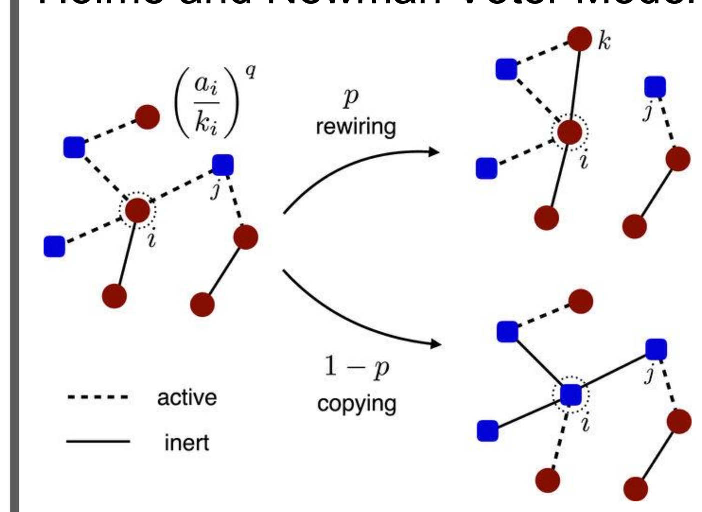
(b)
$$\lambda = \begin{cases} 0, \\ 0, \\ -\frac{N}{K}\frac{1}{1-v^2}[1 - vw + \gamma(v^2 - 1)] \gtrless 0 \quad (\gamma \gtrless 1). \end{cases}$$

The curve is bounded by ±1 along the *v* = *w* axis. As shown by Figure 2, as *u*→1, fixed point (a) becomes a stable attractor. This indicates that all nodes converge to have the same opinion at this fixed point when $\gamma > 1$. However, in Figure 3, when $\gamma < 1$, fixed point (a) disappears within 0 ≤ *u* ≤ 1, and *u* converges at 1 while *v* and *w* do not change their values, and the point is unstable. This indicates that the network will split at this point distinctly into those with opinion A and those with opinion B, shown also by the fact that when *u* = 1, it is because [AB] = 0.

Network Construction

Note: α here is equivalent to ϕ in the system of differential equations

Holme and Newman Voter Model



p = probability of rewiring node with discordant edge to a new node with the same value

1-p = probability of changing values of a node to match node connected with a discordant edge

Active edge = edge between two unlike opinions (discordant edge)

Voter Model Function

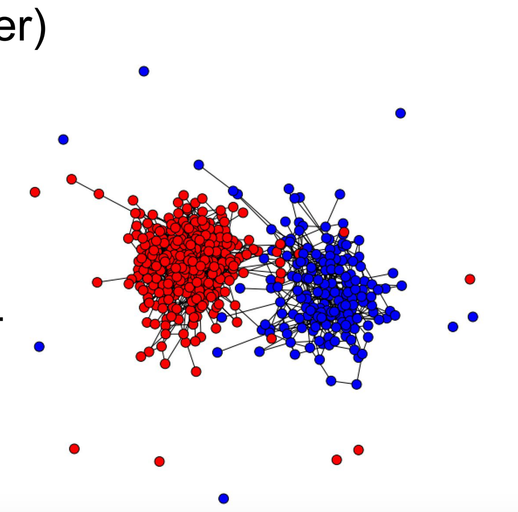
```
def voter_model_step(G0, O, alpha):

    #Defining needed numbers for voter model

    edges_in_G = np.array(G0.edges())
    discordant_edges = dis_calc(G0, O)

    edge_index = random.randrange(len(discordant_edges));
    nL = discordant_edges[edge_index][0];
    nr = discordant_edges[edge_index][1];

    xi = random.uniform(0,1)

    #if over the alpha value nodes change value

    if xi > alpha:
        O[nL] = O[nr]

    #if under the alpha value nodes break connection and form new random connection

    else:
        xn1 = choice(edges_in_G); nL1= xn1[0]; nr1 =xn1[1];
        nr1_value = O[nr1]

        while nr1_value != O[nL]:
            xn1 = choice(edges_in_G); nL1= xn1[0]; nr1 =xn1[1];
            nr1_value = O[nr1]

        G0.remove_edge(nL, nr)
        G0.add_edge(nL, nr1)

    return [G0, O]
```

G0 = random network generated from networkx python package with 500 nodes and 1200 edges on average (initially 600 discordant edges)

α = probability of changing values of a node to match node connected with a discordant edge (parameter)

Visualization of the rewire-to-random model soon before fission occurs for *N* = 500 nodes, alpha value = 0.3, edges number = 1227, and α = 3.0.
Colors correspond to the two opinions (red = pro-choice).

Percent anti-choice: 0.352, percent pro-choice: 0.648
Predicted infant mortality rate: 2.092846144



Network behavior is highly dependent on the initial percentages of public opinion and α. This dependence can be better understood using the Holmes and Newman mathematical model (which uses ϕ = α).

```
def Voter_Model(state, t):
  # unpack the state vector that you input
  x = state[0]
  y = state[1]
  z = state[2]

  # compute state derivatives
  xd_1 = (N/K)*((1.0-x)/(1.0-(y**2.0)))
  xd_2 = 1.0-((y)*(z))+((1.0-2.0*x+(y**2.0))*gamma)

  dx = xd_1*xd_2
  dy = (N/K)*(1.0-2.0*alpha)*((1.0-x)/(1.0-(y**2.0)))*(y-z)
  dz = 2.0*(1.0-alpha)*((1.0-x)/(1.0-(y**2.0)))*(y-z)

  # return the state derivatives in a derivaiive vector

  return [dx, dy, dz]
```

Conclusions/Limitations

After performing a bifurcation on our differential model, we saw the stability for our critical points was replicated in the stability of our network model. From this, we could conclude that our results are consistent with our model. This seems to indicate that the rate of change of public opinion does impact opinion formation, and thus this rate of opinion change regarding reproductive health care is one contributor to the vulnerability to infant mortality. However, our model was limited in its small size (500 nodes), and used a limited data set that was not broken down by racial demographics, something known to also impact vulnerability. For future studies, increasing population size and utilizing more detailed data to build our model may better represent the population on a national level.

Citations:
[1] Chen, Alice, et al. "Why Is Infant Mortality Higher in the United States than in Europe?" *American Economic Journal: Economic Policy*, vol. 8, no. 2, 2016, pp. 89–124., doi:10.1257/pol.20140224.
[2] Infant Mortality Rates by State. (2018, January 11). Retrieved May 20, 2018, https://www.cdc.gov/nchs/pressroom/sosmap/infant_mortality_rates/infant_mortality.htm
[3] Wormald, B. (2015, May 11). Religious Landscape Study: Views about Abortion by State. Retrieved May 20, 2018, http://www.pewforum.org/religious-landscape-study/compare/views-about-abortion/by/state/
[4] Kimura, D., & Hayakawa, Y. (2008). Coevolutionary networks with homophily and heterophily. *Physical Review E*, *78*(1). doi:10.1103/physreve.78.016103