

A Composition Formula for Asymptotic Morphisms (Thesis Defense)

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Abstract

Given graded C^* -algebras A and B , we define the notion of an admissible pair (ϕ, D) for A and B . Associated to an admissible pair (ϕ, D) is an equivalence class of asymptotic morphisms from A to B . Under certain hypotheses, we develop a composition formula for the composition of asymptotic morphisms arising from admissible pairs. Let $\Phi = (\phi, D)$ be an admissible pair for (A, B) and $\Psi = (\psi, E)$ be an admissible pair for (B, C) . We formulate conditions under which $(\psi \circ \phi, \psi(D) + E)$ is an admissible pair for (A, C) and the composition $[[\Psi]]_a \circ [[\Phi]]_a$ is equivalent to $[[\psi \circ \phi, \psi(D) + E]]_a$. We then discuss the perturbation theory of unbounded multipliers of C^* -algebras. In particular, we prove an analogue of the Kato-Rellich theorem for self-adjoint multipliers. This provides a framework to perform the arithmetic with unbounded multipliers needed in the composition formula. We conclude with an application of the composition formula to the stability of K-homology classes of elliptic operators on closed, compact Riemannian manifolds under zeroth order perturbations.