

# Selectivity in central simple algebras and isospectrality

Benjamin Linowitz

## *Abstract*

This thesis investigates the embedding theory of orders in central simple algebras, placing a particular emphasis on the role that the phenomenon known as selectivity plays in the theory. Although the notion of selectivity is completely algebraic, it arose from the study of complex manifolds and provides important insight into various problems arising in differential and hyperbolic geometry. This thesis examines selectivity from a variety of perspectives. Using algebraic number theory we will prove several theorems which clarify when selectivity can and cannot occur. We will then use orders in quaternion algebras to construct discrete groups of isometries acting on products of hyperbolic upper-half planes and upper-half spaces. Within this context we will employ selectivity to provide examples of hyperbolic orbifolds of extremely small volume which are isospectral with respect to the Laplace-Beltrami operator but which are not isometric.