Pattern Avoiding Permutations & Rook Placements

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Abstract

First, we look at the distribution of permutation statistics in the context of pattern-avoiding permutations. The first part of this chapter deals with a recursively defined bijection of Robertson [6] between 123- and 132-avoiding permutations. We introduce the general notion of permutation templates and pivots in order to give a non-recursive pictorial reformulation of Robertson's bijection. This new description in turn makes it obvious that this bijection preserves the number of fixed points and exceedances of a permutation. This confirms a conjecture of Robertson from the 2003 Integers conference. In the last part of Chapter 2 we consider a conjecture of Sagan [4] involving the distribution of descents and patterns of length 4. In particular, we introduce an explicit bijection between 2413- and 1423-avoiding permutations that preserves descents as well as many other permutation statistics. This work generalizes and simplifies Stankova's bijection in [8] which is fundamental to the classification of Wilf-equivalence classes for patterns of length 4.

In Chapter **??** we shift focus and consider an important bijection, due to Backlin, West, and Xin [1], in the context of pattern avoiding rook placements. Using the notion of pivots from Chapter **??**, we prove that this bijection has an equivalent, yet far from obvious, definition in terms of Fomin's growth diagrams. Armed with this new characterization, it becomes clear that this bijection commutes with inverses. This answers a question first posed by Bousquet-Mélou and Steingrímsson [2] and later by Krattentahler [5].

Continuing our study of pattern avoiding rook placements, we study shape-Wilf-equivalence in Chapter 4. Although Stankova and West in [7] first established that 231 and 312 are shape-Wilf-equivalent, their proof is nonbijective and somewhat complicated. Here we give a bijective and elucidating proof of this result, that, besides its simplicity, yields many important enumerative results that we explore in Chapter 5.

In the context of matchings and set partitions we prove several new enumerative results. In particular, we define a notion of pattern-avoiding matchings and set partitions that not only extends the well studied idea of k-crossings and k-nestings but also plays nicely with the concept of shape-Wilf-equivalence. Although 123-avoiding matchings had previously been counted [3] we provide the first enumeration of 231-avoiding matchings and set partitions using the main idea from Chapter 4. Using similar enumerative techniques we also provide a new and simpler proof of Bóna's celebrated formula for the generating function of 2314-avoiding permutations.

References

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