Results on Minimizing Closed Geodesics

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Abstract

In this thesis we study $1/k$-geodesics, those closed geodesics that minimize on any subinterval of length L/k, where L is the length of the geodesic. These curves arise as critical points of the uniform energy, a function introduced in Morse theory as a finite dimensional approximation to the Morse energy function. The uniform energy is defined as a sum of squared distance functions and we therefore complete a detailed study of the differentiability of the Riemannian distance function. We introduce a generalized notion of a critical point for the uniform energy and provide a relationship between these generalized critical points and the $1/k$-geodesics on compact Riemannian manifolds. These generalized critical points are then studied in various different settings, including under Gromov-Hausdorff convergence and in relation to the Grove-Shiohama critical points of distance. We construct surfaces to demonstrate the existence and non-existence of half-geodesics (1/2-geodesics) on manifolds diffeomorphic to S^2.