c-Wilf equivalences of permutations

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Abstract

   In this thesis we study the equivalence relations related to consecutive patterns in permutations. To this end we use the cluster method to prove a new necessary condition for permutations to be strongly c-Wilf equivalent. Also we introduce a new equivalence relation, super strong c-Wilf equivalence, and establish a necessary condition for two permutations to be equivalent under this relation. In particular we show that a known sufficient condition for strong c-Wilf equivalence actually applies to this new relation. We also extend some known results for the non-overlapping permutations. Specifically it was known that c-Wilf equivalence and strong c-Wilf equivalence are the same relation when restricted to the non-overlapping permutations and we show that this is true for super strong c-Wilf equivalence as well. Additionally we give simple relationship between the generating function for permutations with exactly one occurrence of a given non-overlapping pattern and the number of permutations that avoid that pattern. To accomplish these goals we shall use the cluster method of Goulden and Jackson, a scheme for counting permutations according to the positions of occurrences inspired by the work of Billey, Burdzy and Sagan, and a connection between the cluster method to linear extensions of certain posets due to Elizalde and Noy.