On Algebras Of Low Rank And On Belyi Maps

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Abstract

This thesis comes in two parts. The first concerns the classification of algebras of low rank. The main goal of this part is to study the moduli space of rank 3 algebras. Our investigations lead to a generalization of a theorem of Levin that shows that rank 3 algebras over an integral domain occur in essentially two types. We extend this result to rank 3 algebras over a commutative ring, and then to sheaves of rank 3 algebras over a scheme.

In the second part we describe a method for computing Belyi maps. In 1984, Grothendieck described an action of the absolute Galois group of the rational numbers on the set of isomorphism classes of Belyi maps. Thus Belyi maps provide a tantalizing possibility of better understanding this important group. We explain in detail the steps used to compute Belyi maps defined on elliptic and hyperelliptic curves. We conclude with our progress in computing an exhaustive catalogue of Belyi maps, and make some basic observations about the Galois action on these maps.