Pattern Avoidance in Inversion Sequences

Juan S. Auli

Abstract

An inversion sequence of length n is an integer sequence $e = e_1e_2 \dots e_n$ such that $0 \leq e_i < i$, for each i. Corteel–Martinez–Savage–Weselcouch and Mansour–Shattuck began the study of patterns in inversion sequences, focusing on the enumeration of those that avoid classical patterns of length 3. We initiate an analogous systematic study of *consecutive* patterns in inversion sequences, namely patterns whose entries are required to occur in adjacent positions. We enumerate inversion sequences avoiding consecutive patterns of length 3, and generalize some results to patterns of arbitrary length.

We continue this investigation by considering *consecutive patterns of relations*, in analogy to the work of Martinez–Savage in the classical case. Specifically, given two binary relations $R_1, R_2 \in \{\leq, \geq, <, >, =, \neq\}$, we study inversion sequences e with no subindex i such that $e_iR_1e_{i+1}R_2e_{i+2}$. By enumerating such inversion sequences according to their length, we obtain well-known quantities such as Catalan numbers, Fibonacci numbers and central polynomial numbers, relating inversion sequences to other combinatorial structures.

We also study vincular patterns in inversion sequences, which require only certain entries of an occurrence to be adjacent, and thus generalize both classical and consecutive patterns. As in the case of consecutive patterns of relations, by enumerating such sequences according to their length, we also obtain several well-known quantities, including Bell and Fishburn numbers.

Additionally, we classify all vincular patterns of length 3 in inversion sequences—consecutive, classical, and hybrid—as well as consecutive patterns of relations into Wilf equivalence classes, according to the number of inversion sequences avoiding them, and into more restrictive classes that consider the positions of the occurrences of the patterns.