New Techniques in Optimal Transport
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Abstract

This thesis develops a new technique for applications of optimal transport and presents a new perspective on optimal transport through the measure theoretic tool of transition kernels. Optimal transport provides a way of lifting a distance metric on a space to probability measures over that space. This makes the field well suited for certain types of image analysis. Part of this thesis focuses on a new application for optimal transport, while the other focuses on a new approach to optimal transport itself.

With respect to the first part of this thesis, we propose using semi-discrete optimal transport for the estimation of parameters in physical scenes and show how to do so. Optimal transport is a natural setting when studying images because displacements of the objects in the image directly correspond to a change in the optimal transport cost.

In the second part of this thesis we discuss transition kernels, which provide a mathematical tool that can be used to map measures to measures. It therefore seems intuitive to incorporate transition kernels into optimal transport problems. However, this requires changing the traditional perspective of viewing optimal transport as primarily a tool to measure distances between two fixed measures. To that end, this thesis develops theory to show how kernels may be used to extend optimal transport to signed measures.