Quantum Groups and Open Problems in Total Nonnegativity

Neelesh Tiruviluamala
Advisor: Mark Skandera
2005

Abstract

We give an elementary introduction to quantum groups, total nonnegativity and Schur nonnegativity, and consider two open problems in these areas. In particular, we define Hopf algebra duality and include examples of dual pairs known as quantum groups. The elements of one such quantum group, the coordinate ring of $SL_n(\mathbb{C})$, may be thought of as functions on $n \times n$ matrices, and a special basis of this quantum group consists of functions possessing the total nonnegativity and Schur nonnegativity properties. We present an elementary introduction to these properties, including a discussion of Lindstrom's Lemma, and consider two open problems concerning the Murnaghan-Nakayama rule and symmetric functions.