Senior Thesis

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Tools for computing the capacitance of the unit cube and Laplace eigenfunctions on the sphere

Abstract:
Calculating the capacitance of a unit cube requires the numerical solution of a Laplace boundary value problem (BVP). However, current methods observe a slow algebraic error convergence. In this paper, we suggest the method of particular solutions (MPS) with domain decomposition technique to the BVP, which can handle the corner singularity of the cube and is spectrally accurate. The 2d analogous problem of calculating the capacity of the unit square with MPS is demonstrated, and for the 3d problem we focus on accurate methods to compute the needed basis functions based at the corner of the cube. This involves solving a Dirichlet boundary eigenvalue problem on 7/8 of a sphere. Fast algorithms for evaluating the required associated Legendre function are also discussed. We have obtained 15 digits accuracy in computing the capacity of the unit square with an exponential error convergence. For the related Dirichlet eigenvalue problem, we got 5 digits accuracy for the eigenvalues on the sphere.