The Kronecker Product of Two Monomial Symmetric Functions

Katherine Roddy

Abstract

This thesis investigates the problem of computing the Kronecker product of two monomial symmetric functions. Our main result is a formula that can be easily programmed to compute this product for any two monomial symmetric functions in commuting variables. The main tool in obtaining this formula is the algebra of symmetric functions in non-commuting variables. This algebra has basis elements that are indexed by set partitions. Hence, the formulas we obtain are in terms of the M¨obius function on the lattice of set partitions. A combinatorial interpretation for the coefficients of the Kronecker product of two monomial symmetric functions was given by Remmel and Whitehead in [1] in terms of an object called a primitive bi-brick cycle. In this thesis, we provide a formula for enumerating these objects and give alternate proofs for several of Remmel and Whitehead's results. In particular, we can characterize the sign of the coefficients as well as when the coefficient is zero. Further, we give formulas for transition matrices between the monomial symmetric functions and the basis of power sum symmetric functions in commuting variables. These change of basis matrices are in terms of the M¨obius function on the lattice of set partitions. By using these transition matrices, we are able to produce another formula for the coefficients of the Kronecker product of two monomial symmetric functions. Finally, we give several explicit formulas for special cases of the product.

[1] J.B. Remmel and T. Whitehead, *Transition matrices and kronecker product expansions of symmetric functions*, Linear Multilinear Algebra 40 (1996), 337–352.