a crash course in game theory

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A brief history of game theory

- Von Neumann and Morgenstern's 1944 book, the *Theory of Games and Economic Behavior,* marked a major advancement in the field of game theory. [1]
- Economists would begin using this framework to understand **rational decision-making** among players which relies on a core set of assumptions: [2]

    1. Decision-makers pursue well-defined objectives and are **rational.**

    2. Decision-makers take into account other decision makers' behavior and **reason strategically.**

- Today, game theory is not only used by economists but also social scientists, biologists, and policy researchers.

Game theory’s basic mathematical formulation

- **Game:** the situation *N* players find themselves in, regulated by rules and pay-offs.
  - **Strategic games:** games in which players simultaneously make one decision, modeled with three parameters, $< N, (A_i) , (\gtrsim_i) >$.
  - **Extensive games:** games in which players iteratively make decisions, modeled with four parameters, $< N, H , P , (\gtrsim_i) >$.

- **Game properties and deviations:**
  - Constant-sum VS. variable-sum games
    - *Zerosum* games (strictly competitive games)
  - Cooperative VS. non-cooperative games
  - Bargaining games
  - Common VS hidden / imperfect knowledge
  - Deterministic VS. non-deterministic games

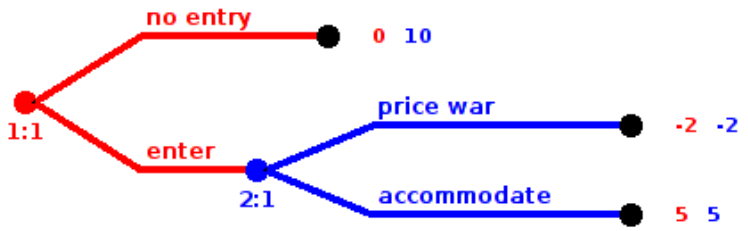
Game theory examples

Strategic game example: Prisoner's Dilemma

	Confess	Refuse
Confess	2,2	4,0
Refuse	0,4	3,3

$< N, (A_i), (\gtrsim_i) >$

Extensive game example: chain-stores



$< N, H, P, (\gtrsim_i) >$

“Winning” in game theory

- “Winning” in game theory = maximizing your own payoff with the assumption that the other players are rational and reason strategically to achieve their own best payoff.
- **Nash equilibrium profile *a**:**

  the profile $a^* \in A$ of actions with the property that for every player $i \in N$, we have $(a_{-i}^*, a_i^*) \succsim_i (a_{-i}^*, a_i)$ for all $a_i \in A_i$

  - the profile of actions describing the steady state achieved in a game when no player can further maximize their own payoff.
  - “Each action in a Nash equilibrium is a best response to the other actions.”
- **Subgame perfect Nash Equilibrium profile *s**:** a more practical application of the Nash equilibrium to extensive games.

What would you do?

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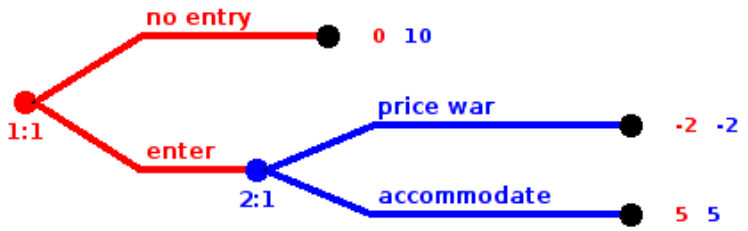
Extensive game example: chain-stores



Practicality of game theory: the players and the games

societal and interpersonal relationships

international relations and diplomacy

individual - institution relationships

How an economist helped thousands get a new kidney

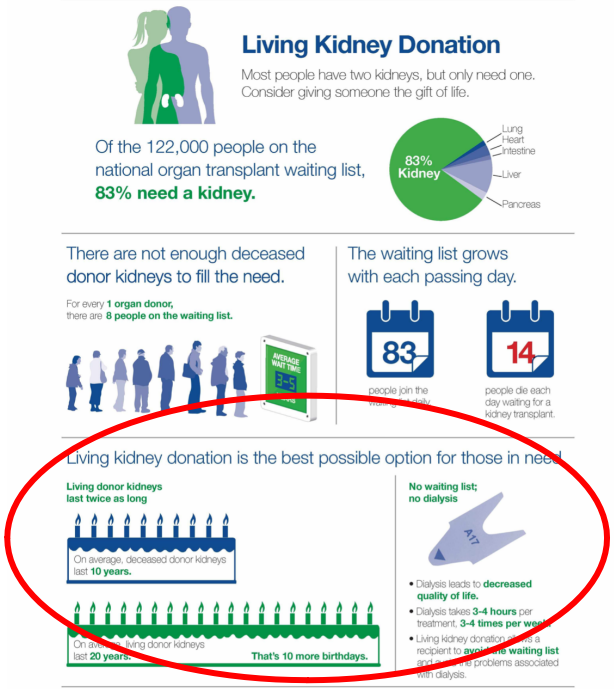
By Ian Rose, BBC News
Berlin

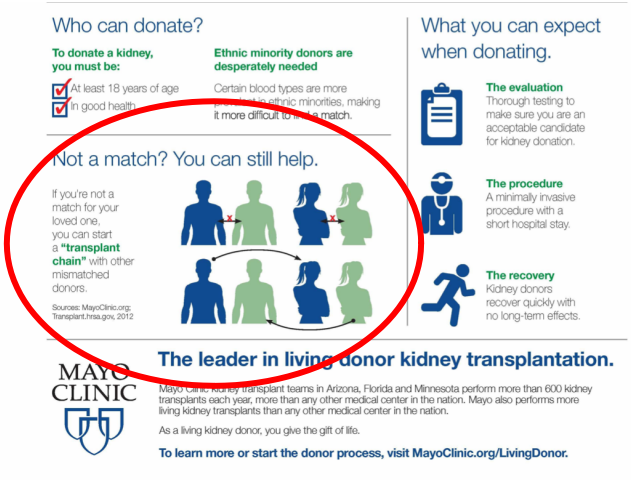
17 December 2019

Case study: the U.S. Kidney Exchange Program

- A supply-demand problem with organ transplants in the US exists.
  a. Note that the **selling of human organs** is illegal.
  b. Also note that the blood and tissue type of both the donor and the recipient matters**.**
  c. Lastly, note that **almost all organs** in the human body serve an irreplaceable purpose.
     - The one exception relates to the kidneys.

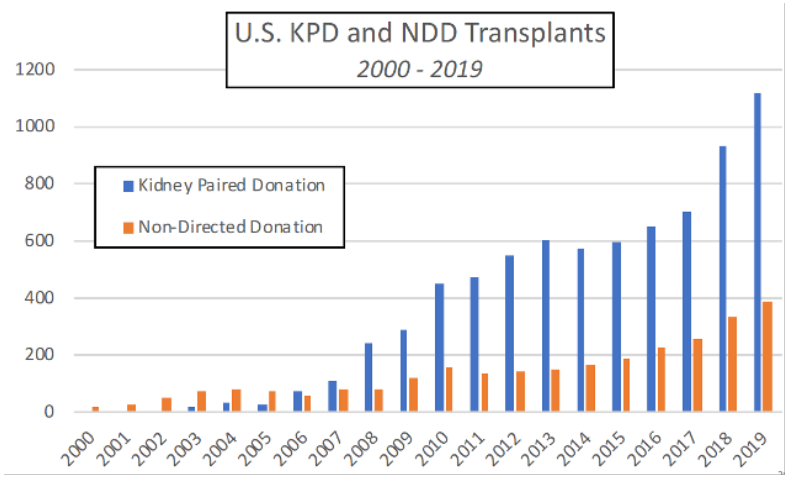
Case study cont.



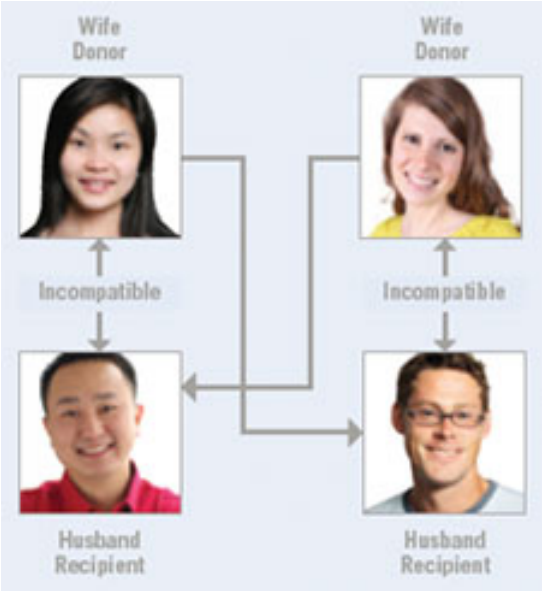


[Infographic from the Mayo Clinic]

Case study cont.

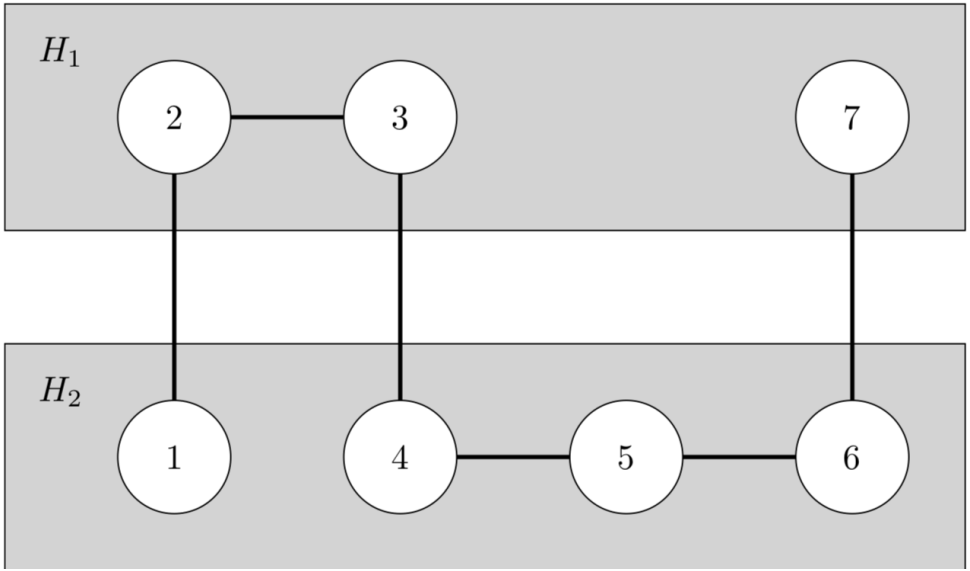


[Microeconomic Insights]



[UCLA Kidney Swap]

Game theory analysis: how to incentivize full reporting?



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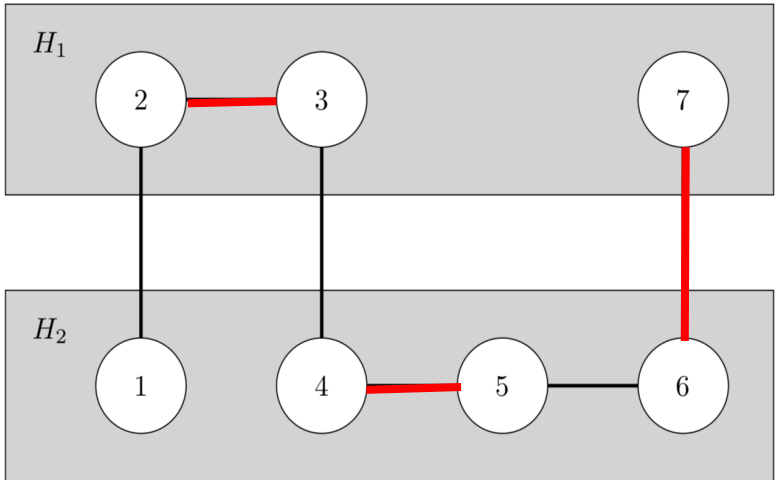
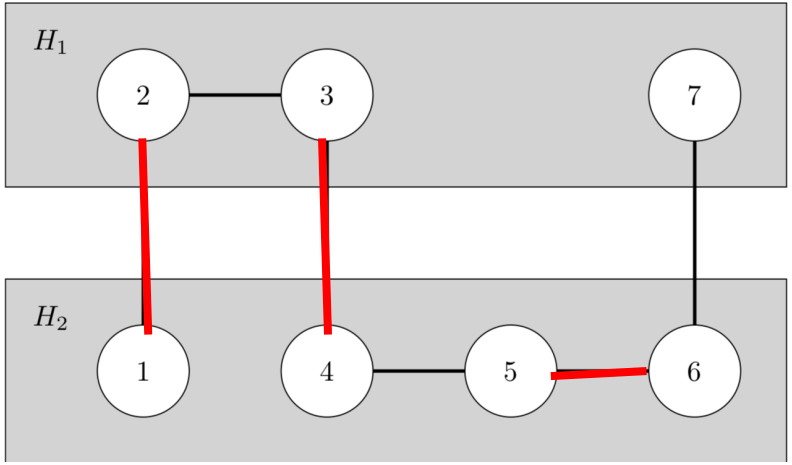


Hospital 1 lies by omission.



Hospital 2 lies by omission.

Conclusion

1. **Game theory** is the analysis of **games** and **solutions**, otherwise known as the analysis of specific circumstances involving a certain number of players, a designated profile of actions, and a potential set of pay-offs.
2. Game theory started off as an economics concept and has since been embraced by other fields such as psychology, sociology, and biology.
    a. It can also be more formally modeled and understood with mathematics.
3. The two most well-known types of games are **strategic** games and **extensive** games, described by their optimal **Nash equilibrium** or **subgame perfect Nash equilibrium,** respectively.
4. Game theory is important and has been adopted by so many other fields because it attempts to understand and characterize the interactions that make up society.
5. One example of game theory in action are the matching market innovations of the late 90s and early 2000s, led by Alvin Roth (NRMP and kidney exchange programs).
6. Game theory can be called upon to analyze systems and redesign them in ways to encourage compliance and fair-play.

Citations

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Thank you!

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