MATH 101: ALGEBRA I
HOMEWORK, DAY #19

Let R be a ring and let M be a (left) R-module.

Problem JV19.A. An element $m \in M$ is called a *torsion element* if $rm = 0$ for some nonzero $r \in R$. The set of torsion elements is denoted $\mathrm{Tor}(M)$.

(a) Prove that if R is an integral domain, then $\mathrm{Tor}(M)$ is a submodule of M.
(b) Give an example of a ring R and an R-module M such that $\mathrm{Tor}(M)$ is not a submodule.
(c) Show that if R has a zerodivisor then every nonzero R-module M has $\mathrm{Tor}(M) \neq \{0\}$.
(d) M is called a *torsion module* if $M = \mathrm{Tor}(M)$. Prove that every finite abelian group is a torsion $\mathbb{Z}$-module. Give an example of an infinite abelian group that is a torsion $\mathbb{Z}$-module.

Date: Assigned Wednesday, 12 October 2016; due Friday, 14 October 2016.