MATH 101: ALGEBRA I
HOMEWORK, DAY #31

Problem JV31.A. Let R be a PID and let M be a finitely generated torsion R-module. Show that there exists $y \in M$ such that $\operatorname{Ann}(y) = \operatorname{Ann}(M)$.

Problem JV31.B. Let M be the $\mathbb{Z}$-module generated by x_1, x_2, x_3, x_4 subject to the relations

$$\begin{aligned} x_1 + 3x_2 - 9x_3 \qquad\quad &= 0 \\ x_1 + 3x_2 + 3x_3 + 12x_4 &= 0 \\ 2x_1 + 4x_2 + 2x_3 + 24x_4 &= 0 \end{aligned}$$

Give an explicit isomorphism of M to a direct sum of cyclic abelian groups. What are the invariant factors and elementary divisors of $\operatorname{Tor}(M)$?

Date: Assigned Monday, 7 November 2016; due Tuesday, 8 November 2016.