MATH 101: GRADUATE LINEAR ALGEBRA
DAILY HOMEWORK #8

Problem 8.1. Let $F = \mathbb{R}$ or $F = \mathbb{C}$ and let V, W be finite-dimensional inner product spaces over F (i.e., vector spaces equipped with an inner product). Let $T \colon V \to W$ be an F-linear map, and let $T^* \colon W \to V$ be the adjoint with respect to the inner product, so that $\langle T(v), w \rangle_W = \langle v, T^*(w) \rangle_V$ for all $v \in V$ and $w \in W$.

Let $v \in V$. Show that $(T^*T)(v) = 0$ if and only if $T(v) = 0$.