MATH 101: GRADUATE LINEAR ALGEBRA
DAILY HOMEWORK #9

Problem 9.1. Let $F = \mathbb{R}$ or $F = \mathbb{C}$, let V be a finite-dimensional inner product space over F. Let $\phi\colon V \to V$ be a linear operator.

(a) Let ϕ be self-adjoint, and suppose that $\langle x, \phi(x)\rangle = 0$ for all $x \in V$. Show that $\phi = 0$.

(b) Suppose that $\|\phi(x)\| = \|x\|$ for all $x \in V$. Show that $\phi\phi^* = \phi^*\phi = 1$. *[Hint: Apply part* (a) *to* $\phi^*\phi - 1$*.]*

Date: Assigned Monday, 2 October 2017; due Wednesday, 4 October 2017.