Problem 11.1. Let V be an F-vector space, and let $\phi\colon V \to V$ be an F-linear map. We say that ϕ is an *involution* if $\phi^2 = 1$.

Suppose that V is an inner product space and let ϕ be an involution on V. Show that the following are equivalent:

(a) ϕ is normal;
(b) ϕ is unitary;
(c) ϕ is self-adjoint; and
(d) All singular values of ϕ are equal to 1.