MATH 101: GRADUATE LINEAR ALGEBRA
DAILY HOMEWORK #18

Problem 18.1. Let R be a (commutative integral) domain. An R-module A is *divisible* if $rA = A$ for every nonzero $r \in R$.

Let Q be a nonzero divisible $\mathbb{Z}$-module. Prove that Q is *not* a projective $\mathbb{Z}$-module. Deduce that $\mathbb{Q}$ is not a projective $\mathbb{Z}$-module. *[Hint: Show first that if F is a free module then $\bigcap_{n=1}^{\infty} nF = \{0\}$ using a basis. Suppose that Q is projective, and use one of the equivalent conditions on a projective module.]*

Problem 18.2. Let R be a commutative ring. Let M, N be projective R-modules. Show that $M \otimes_R N$ is a projective R-module. *[Hint: Use that the tensor product of two free R-modules is free, because tensor products commute with direct sums.]*

Date: Assigned Friday, 20 October 2017; due Monday, 23 October 2017.