20. How many $\mathbb{Z}$-bilinear maps are there from $\mathbb{Z} \times \mathbb{Z}$ to $G$, where $G$ is any finite abelian group? Describe them explicitly.

21. Is it possible to define a multiplication which makes the additive group $\mathbb{Q}/\mathbb{Z}$ into a ring?

22. Show that, in general, $M \otimes_{\mathbb{Z}} N \not\approx M \otimes_R N$, but that there is a surjection form one of these groups to the other. Describe, in a specific example, a nontrivial element of the kernel of this homomorphism.

23. Show that tensor products do not commute with products in general. Hint: Consider $\prod_i \frac{\mathbb{Z}}{(2^i)} \otimes \mathbb{Q}$.

24. Let $V$ be a finite-dimensional $k$-vector space.

    (a) Show that there is a linear transformation $T\colon V \otimes_k V^* \to k$ defined by $T(v \otimes \varphi) = \varphi(v)$.

    (b) The contraction $T$ corresponds to a linear transformation $\tau\colon \operatorname{End}_k(V) \to k$ via the isomorphism $V \otimes_k V^* \to \operatorname{Hom}_k(V, V) = \operatorname{End}_k(V)$:

    $$\begin{array}{ccc} V \otimes_k V^* & \xrightarrow{\approx} & \operatorname{End}_k(V) \\ & {\scriptstyle T} \searrow & \downarrow {\scriptstyle \tau} \\ & & k \end{array}$$

    What familiar linear map is $\tau$?