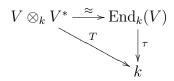
- 20. How many \mathbb{Z} -bilinear maps are there from $\mathbb{Z} \times \mathbb{Z}$ to G, where G is any finite abelian group? Describe them explicitly.
- 21. Is it possible to define a multiplication which makes the additive group \mathbb{Q}/\mathbb{Z} into a ring?
- 22. Show that, in general, $M \otimes_{\mathbb{Z}} N \not\approx M \otimes_R N$, but that there is a surjection form one of these groups to the other. Describe, in a specific example, a nontrivial element of the kernel of this homomorphism.
- 23. Show that tensor products do not commute with products in general. Hint: Consider $\prod_i \frac{\mathbb{Z}}{(2^i)} \otimes \mathbb{Q}$.
- 24. Let V be a finite-dimensional k-vector space.
 - (a) Show that there is a linear transformation $T: V \otimes_k V^* \to k$ defined by $T(v \otimes \varphi) = \varphi(v)$.
 - (b) The contraction T corresponds to a linear transformation $\tau \colon \operatorname{End}_k(V) \to k$ via the isomorphism $V \otimes_k V^* \to \operatorname{Hom}_k(V,V) = \operatorname{End}_k(V)$:



What familiar linear map is τ ?