Homework Assignment #3
Due Wednesday, February 14th

1. Recall that a set C in a vector space X is called convex if $x, y \in C$ and $\lambda \in [0,1]$ implies that $\lambda x + (1-\lambda)y \in C$.

(a) Suppose that $x_1, \dots, x_n \in X$. If $\lambda_i \geq 0$ and $\sum_{i=1}^n \lambda_i = 1$, then $\sum_{i=1}^n \lambda_i x_i$ is called a convex combination of the x_i. Show if C is convex, then any convex combination of elements from C belongs to C.

(b) Show that if C is a convex subset of a topological vector space X, then its closure, $\overline{C}$ is also convex.

(c) Work problem E 2.4.1 in the text.

2. Let $\{ F_j \}_{j \in J}$ be a collection of nonempty closed subsets in a compact space X which is totally ordered by reverse containment.[1] Then

$$\bigcap_{j \in J} F_j \neq \emptyset.$$

(Hint: consider the complement.)

3. Suppose that X is a compact topological space and that $f : X \to \mathbf{R}$ is continuous. Show that f attains its maximum and minimum on X; that is, show that there are points $y, z \in X$ such that

$$f(y) \leq f(x) \leq f(z) \quad \text{for all } x \in X.$$

(Hint: use Theorem 1.6.2(v).)

4. Work E 2.4.5.

5. Work E 2.4.6.

6. Work E 2.4.16 in the "Revised Printing" of the text. If you don't have access to the revised printing, email me, and I'll send you a pdf of the relevant problem page.

[1] Recall that "ordered by reverse containment" simply means that $F_j \geq F_{j'}$ if and only if $F_j \subset F_{j'}$.

7. Work E 2.4.17 in the “Revised Printing” of the text. (Warning, this problem is a bit involved. You may consider it “extra credit” if that helps. Nevertheless, I though it was interesting.)