Homework Assignment #3
Due Wednesday, February 3rd

Instructions: As usual, for the "true/false" questions, just circle the correct answer. No justifications are required, but don't guess. You score is based on #right minus #wrong.

1. **TRUE or FALSE**: The dual of any normed vector space is a Banach space.

2. **TRUE or FALSE**: If X and Y are Banach spaces and $T : X \to Y$ is a surjective linear map, then T is bounded.

3. **TRUE or FALSE**: If Y is a closed subspace of a normed vector space X and if $x \in X \setminus Y$, then there is a $\varphi \in X^*$ such that $\varphi(y) = 0$ for all $y \in Y$ and $\varphi(x) = 1$.

4. **TRUE or FALSE**: Suppose that X and Y are Banach spaces and that $T_n : X \to Y$ is a bounded linear map for $n = 1, 2, 3, \ldots$. Suppose that there is a linear operator $T_0 : X \to Y$ such that for each $x \in X$, we have $T_n x \to T_0 x$. Then T is bounded.

5. Suppose that Y is a subspace of a normed vector space X. Show that the closure of Y is given by

$$\overline{Y} = \bigcap \{\, \ker \varphi : \varphi \in X^* \text{ and } Y \subset \ker \varphi \,\}.$$

6. Work E.2.3.2 in the text. If may be helpful to think of c_0 as $C_0(\mathbf{N})$. Then if $x \in C_c(\mathbf{N})$, we have $x = \sum x_n \delta_n$, where the x_n are scalars and δ_n is the function taking the value 1 at n and 0 elsewhere.

7. Work E.2.3.4 in the text.

8. Work E.2.3.5 in the text.

9. Work E.2.3.7 in the text.