Homework Assignment #6
Due Wednesday, March 10th.

1. Let $\{ e_j \}_{j \in J}$ be an orthonormal basis for $\mathcal{H}$. Show that U is unitary if and only if $\{ Ue_j \}_{j \in J}$ is an orthonormal basis for $\mathcal{H}$.

2. Suppose that P and Q are projections in $B(H)$. We say that $P \perp Q$ if $P(H) \perp Q(H)$ and that $P \leq Q$ if $P(H) \subset Q(H)$.

(a) Show that the following are equivalent.

(i) $P \perp Q$.

(ii) $PQ = QP = 0$.

(iii) $P + Q$ is a projection.

(b) Show that the following are equivalent.

(i) $P \leq Q$.

(ii) $PQ = QP = P$.

(iii) $Q - P$ is a projection.

(Hint: Note that PQP is a positive operator. Also $PQP = PQ(PQ)^*$ so that $PQP = 0$ if and only if $PQ = QP = 0$.)

3. Work E 3.3.1 in the text.

4. Work E 3.3.2 in the text.

5. Work E 3.3.4 in the text.