MATH 116 WORKSHEET : Companion matrix

Barnett
3/20/14

SOLUTIONS

A) Compute $\det\begin{pmatrix} z & a_2 \\ -1 & a_1+z \end{pmatrix} = z^2 + a_1 z + a_2$

B) Compute $\det\begin{pmatrix} z & & a_3 \\ -1 & z & a_2 \\ & -1 & a_1+z \end{pmatrix}$ reusing the above.

$= z(z^2 + a_1 z + a_2) + (-1)^2 a_3$

$= z^3 + a_1 z^2 + a_2 z + a_3$

ie general monic poly.

C) A 'monic polynomial' has leading coefficient 1.
Show how to build a matrix whose eigenvalues are the roots of a general monic (& hence non-monic) polynomial of degree p :

since can divide coeffs by the leading coef.

[this is called 'companion matrix']

eigval prob has to be: $\det\begin{bmatrix} z & & & & & a_n \\ -1 & z & & & & a_{n-1} \\ & -1 & z & & & a_{n-2} \\ & & -1 & z & & \vdots \\ & & & -1 & \ddots z & a_2 \\ & & & & -1 & a_1+z \end{bmatrix} = 0$ so $A = \begin{bmatrix} 0 & 0 & 0 & \cdots & -a_n \\ 1 & 0 & 0 & \cdots & -a_{n-1} \\ 0 & 1 & 0 & \cdots & \vdots \\ & & 1 & \ddots & -a_2 \\ & & & 1 & -a_1 \end{bmatrix}$

here.

has rts as eigvals.

D) So, can there be a direct algorithm for the $n \geq 5$ general matrix EVP?

No, since if there were, one could use it (as above) to give direct exact formula for roots of quintic & higher polynomials, which by Galois, Abel, etc 1824, we know is impossible!

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