Math 13 Fall 2009 Homework 9 DUE WEDNESDAY DECEMBER 2ND IN CLASS

1.) Explain how Stoke's Theorem is a generalization of Green's Theorem. That is, why does the statement of Stoke's Theorem include Green's Theorem as a special case? As a suggestion, try proving Green's Theorem assuming Stoke's Theorem as a given.

2.) If S is the sphere centered at $(\pi, \pi + 10, 10\pi + 100)$ of radius $1000e^{\pi}$, and $\mathbf{F}(x, y, z) = (e^{x^{\pi}+97ye^{z}}, e^{e^{xyze^{z}}}, \pi)$, compute

$$\int\int_S \mathrm{curl}\mathbf{F} \cdot \mathbf{n} dS$$

You are not permitted to ask anyone for help on this except from your the professor. In particular, Asa Levi is not permitted to assist with this problem.

3.) A vector field $\mathbf{F}$ is called *irrotational* at a point (x_0, y_0, z_0) if $\mathrm{curl}\mathbf{F}(x_0, y_0, z_0) = \mathbf{0}$.

a.) Show that the vector field $\mathbf{F}(x, y, z) = (e^z y, e^z x, e^z xy)$ is irrotational for all (x, y, z) in $\mathbb{R}^3$.

b.) Prove that in general, any vector field on $\mathbb{R}^3$ which is the gradient of a scalar function is irrotational. That is, prove that for any vector $\mathbf{F}$ such that $\mathbf{F} = \nabla f$ for a scalar function $f : \mathbb{R}^3 \to \mathbb{R}$, the vector field $\mathbf{F}$ is irrotational.

c.) Re-answer part (a) using only one sentence.

d.) Show that any vector field of the form $\mathbf{F}(x, y, z) = (f(x), g(y), h(z))$ is irrotational.

e.) Compute

$$\int_C \mathbf{F} \cdot \mathbf{T} ds$$

where $\mathbf{F}(x, y, z) = (x^2+2x+\sin(\cos(x)), e^{\cos(y)}y^{100}+y^e, z^{\pi}+\cos(\sin(\cos(z))))$, and C is parameterized by $\mathbf{r}(t) = (\cos(t), \sin(t), \sin(t))$ on the interval $0 \leq t \leq 2\pi$.