GREENKEY READING

There are a few important things I did not get to in class today. Please peruse this over the weekend. Feel free to ask me questions about it.

1. I can compute Div and Curl, but I still can't dance to it.

If the above describes you, then hopefully this will help.

In "real life," whatever that is, there are two distinct ways in which Div and Curl are manifest.

1.1. The First Way.

The first is simply the the idea of flow and accumulation [for div] and circulation (for curl).

In particular, if we have a function that represents to the flow of material, a region that have zero divergence is akin to a region in which there is no net flow outward...that is to say however much is going in is going out. This is the conceptual idea behind the "second application" of Stoke's theorems.

In class we said "If div=0, then the total flux through a closed surface must be zero, that is:" $\iint_{S_1} F \cdot \mathbf{dS} + \iint_{S_2} F \cdot \mathbf{dV} = \iiint_V \nabla \cdot \mathbf{F} dV = 0$ **[Where** S_1 **is the hemisphere,** S_2 **is the disk that closes it up, and** V **is the filled in region enclosed by the two.]**

From this we had that $\iint_{S_1} \mathbf{F} \cdot \mathbf{dS} = -\iint_{S_2} \mathbf{F} \cdot \mathbf{dS}$**.**

The "real life" understanding of this is that, since there is no net flow out of the container, however much is flowing <u>out</u> of the top is equal to whatever is flowing <u>into</u> the bottom. [it is "into" because of the negative sign].

The above describes what happens when Div=0 [and the equivalent occurs when Curl=0, but it easier to see in terms of real life with the Div.] But we saw the modified second version could be used when the Div is not 0 we can still say:

$$\iint_{S_1} F \cdot \mathbf{dS} = -\iint_{S_2} F \cdot \mathbf{dV} = \iiint_V \nabla \cdot \mathbf{F} dV$$

This "in real life" means "the amount that goes out the top is equal to the amount that goes in the bottom PLUS whatever amount is created inside."

That is to say that Div(F) in some sense represents the things that cause events. For example, if F **was the flow of water, then Div F would represent a place where**

the water was actually being produced. So if you had a spring, or water welling up from the ground without going back into it, that would be represented by a non-trivial Div.

A similar thing can be said about curl. Places where the curl is non-zero represent something causing rotary motion.

1.2. Second Way. **The second way to think about curl or div is in terms of physics. This is not "required" info, but just a way to maybe get a hold on what is going on. In this case $DivF$ represents charge density. That is to say that if you had a chunk of charged metal, and the metal was giving rise to an electric field, then the div would indicate how dense the charge is [per volume]. This makes sense in terms of the first section, since charge is something "makes things happen."**

A similar phenomenon can be said for curl. A changing magnetic field will cause an electrical current [and a moving current will cause a magnetic field]. In each case the curl in some sense represents the direction and size of the magnetic flux/current. That is to say that the changing magnetic field [or the current] is the thing that is making the other thing happen....so the curl of the effect gives an indication of the size/direction of the the thing causing it. [It is much easier to think about div giving charge/volume, so maybe just stick with that.]

2. Poles and holes

The way a pole [that is a point representing a vector of infinite size], which must correspond to a "hole" in the region, corresponds to this is that a pole represents a concentrated point charge, or a concentrated point mass, or [in today's example] a line of current. That is an example where the charge/current/thing that is causing the effect/ is "smushed" to a point, line, or plane. Because all of the "stuff" that is causing the force is located at a particular point, the total effect [read "line integral" or "surface integral depending on situation] is dependent only upon whether that point/line/plnae is in the region in question or not.

Now, you can have a situation where you have a pole at a point, but that everywhere else the div is not 0 [or the curl is not $\langle 0,0,0\rangle$...In these cases the pole still represents a point charge, but the rest represents a "soup" or charge density....and then you have effects caused by both.

For example $\mathbf{F}(x, y, z) = \langle \frac{x}{x^2+y^2+z^2}, \frac{y}{x^2+y^2+z^2}, \frac{z}{x^2+y^2+z^2} \rangle$ **is an example of a function that has pole at** $\langle 0, 0, 0 \rangle$**.** $Div(F) = \frac{1}{x^2+y^2+z^2}$**, so that means to set up the force associated with** F**, you would not only need a point charge at** $\langle 0, 0, 0 \rangle$**, but you also need more charge spread out with density** $\frac{1}{r^2}$**.**

In non-physics terms, this vector field is spreading out at all points, but at the center it is exploding [it has "infinite" spread-out].