# Math 13

May 2005
Second Practice Midterm

NAME (Print!): ____________________________________

| Problem | Points | Score |
|---|---|---|
| 1 | 5 | |
| 2 | 5 | |
| 3 | 5 | |
| 4 | 5 | |
| 5 | 5 | |
| 6 | 5 | |
| 7 | 5 | |
| 8 | 5 | |
| 9 (Take home) | 25 | |
| 10 (Take home) | 20 | |
| 11 (Take home) | 15 | |
| Total | 100 | |

On the real exam, these will all be multiple choice. For the added challenge on this practice exam, I'm just asking the questions without choices. You should think about what things I might have written for choices.

**Problem 1 (5 points):** Changing the order of integration in

$$\int_0^1 \int_{y^2}^{2-y} f(x,y)\,dx\,dy$$

gives what integral (you may have to write this as the sum of two integrals).

**Problem 2 (5 points):** Sketch the solid whose volume is given by

$$\int_0^{2\pi} \int_0^2 \int_0^{4-r^2} r\,dz\,dr\,d\theta.$$

**Problem 3 (5 points):** Sketch the solid whose volume is given by

$$\int_1^3 \int_0^{\frac{\pi}{2}} \int_r^3 r dz\,d\theta\,dr.$$

**Problem 4 (5 points):** Sketch the solid whose volume is given by

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta.$$

**Problem 5 (5 points):** Sketch the solid whose volume is given by

$$\int_0^{\frac{\pi}{3}} \int_0^{2\pi} \int_0^{\sec\phi} \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi.$$

**Problem 6 (5 points):** Two of the integrals in Problems 2 through 5 can be evaluated without integration. Evaluate them using geometry.

**Problem 7 (5 points):** Is the following true or false? If it's true prove it, if it's false give a counter example: Suppose $f : \mathbb{R}^3 \to \mathbb{R}$ and $F$ is a vector field on $\mathbb{R}^3$. Then

$$\operatorname{div}(f\mathbf{F}) = f\operatorname{div}\mathbf{F} + \mathbf{F} \cdot \nabla f.$$

**Problem 8 (5 points):** Is the following true or false? If it's true, prove it, if it's false, give a counter example: Suppose $F$ is a vector field on $\mathbb{R}^3$. Then $\operatorname{curl}(F) = 0$.

**Problem 9 (20 points):** Find the mass and the center of mass of the lamina that occupies the region $D$ bounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant if the denisty function is $\delta(x, y) = y$.

**Problem 10 (20 points):** Express the integral $\int\int\int_W f(x, y, z)\, dV$ in six different ways where $W$ is bounded by $x^2 + z^2 = 4$, $y = 0$ and $y = 6$ and is expressed in rectangular co-ordinates.

**Problem 11 (20 points):** Find the following two volumes:

(a) The solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.