6.1

12) $\int_\sigma F\cdot ds = \int_0^2 (2t,1,0)\cdot(1,6t,0)\,dt = \int_0^2 8t\,dt = 16$

22) Force has direction $(24, 32-14t)$

$$W = -\int_\sigma F\cdot ds = \int_0^1 25\,\frac{(24,\,32-14t)}{\sqrt{24^2+(32-14t)^2}}\cdot(0,-14)\,dt$$

$$= (-7)(25)\int_0^1 \frac{32-14t}{\sqrt{400-224t+49t^2}}\,dt = -250.$$

23) $\sigma(t) = (t, f(t))$ $\quad\sigma'(t) = (1, f'(t))$.

Then

$$\int_c F\cdot ds = \int_a^b F(\sigma(t))\cdot\sigma'(t)\,dt$$

$$= \int_a^b f(t)\,dt \quad \text{since} \quad F = y\hat{\imath}.$$

6.2

5(a) $\oint y^2dx + x^2dy = \iint_D 2x-2y\,dA$

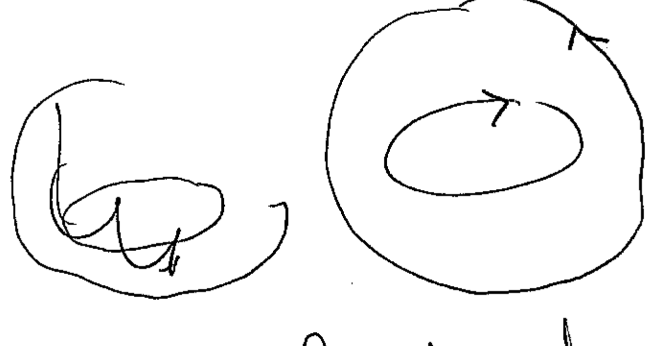
$= \int_0^1\int_0^1 (2x-2y)\,dx\,dy = \int_0^1 1-2y\,dy = 0.$

(b) $C = \begin{cases} \sigma_1(t) = (t,0) & 0\le t\le 1 \\ \sigma_2(t) = (1,t) & 0\le t\le 1 \\ \sigma_3(t) = (1-t,1)\,dt \\ \sigma_4(t) = (0,1-t). \end{cases}$

$\oint_C y^2dx + x^2dy = \int_{\sigma_1} 0\,dt + \int_{\sigma_2} 1\,dt + \int_{\sigma_3} -1\,dt + \int_{\sigma_4} 0\,dt$

$= 1-1=0.$

12) D :



orient this way.

$\sigma_1(t) = (5\cos t, 5\sin t)$

$\sigma_2(t) = (3\cos t, -2\sin t)$

"comes from orientation

Then $\iint_D dA = \frac{1}{2}\oint -y\,dx + x\,dy$

$= \frac{1}{2}\int_0^{2\pi} -5\sin t\cdot(-5\sin t) + 5\cos t\cdot 5\cos t$

$+\frac{1}{2}\int_0^{2\pi} 2\sin t\,(-3\sin t) + 3\cos t\,(-2\cos t)\,dt$

$= 19\pi$

6.2

17 If D is the region bounded by C

$$\oint 3x^2y\,dx + x^3dy = \iint_D 3x^2 - 3x^2\,dA = 0.$$

6.3

12 G: $\frac{\partial N}{\partial x} = 2x = \frac{\partial M}{\partial y}$ $\quad \frac{\partial P}{\partial x} = 0 = \frac{\partial M}{\partial z}$

$\frac{\partial P}{\partial y} = 2y = \frac{\partial N}{\partial z}$ $\quad \therefore$ $G = (2xy, x^2 + 2yz, y^2)$

$G = \nabla(x^2y + y^2z)$

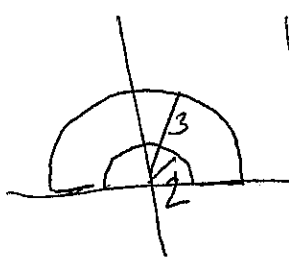
On the other hand F is not conservative since

$$\frac{\partial M}{\partial y} = 2xyz^3 \neq \frac{\partial N}{\partial x} = 4xy.$$

15 $\frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2}}\right) = \frac{-xy}{(x^2+y^2)^{3/2}} = \frac{\partial}{\partial y}\left(\frac{x}{\sqrt{x^2+y^2}}\right).$

So F is conservative if we restrict our domain $(x,y) \neq (0,0)$

D =



Let $\sigma(t) = (2\cos t, 2\sin t)$ $0 \le t \le \pi$

$$\int_C \frac{x\,dy + y\,dx}{\sqrt{x^2+y^2}} = \int_0^{\pi} \frac{4\cos^2 t - 4\sin^2 t}{4\cos^2 t + 4\sin^2 t}\,dt$$

$$= \int_0^{\pi} \cos^2 t - \sin^2 t\,dt$$

$$= \int_0^{\pi} \cos 2t\,dt$$

$$= 0.$$

Could also use Green's thm with D as above.

6(c) We need to reverse the orientation to use green's Thm:

12/ C_1 : $(0,0) \mapsto (0,1) \mapsto (1,0)$

C_2 : opposite direction

D : region bounded by C_1

$$\oint_{C_1} x^2y\,dx + (x+y)y\,dy = -\oint_{C_2} x^2y\,dx + (x+y)y\,dy$$

$$= -\iint_D y - x^2\,dA$$

$$= -\int_0^1\int_0^{1-x} (y-x^2)\,dy\,dx = \frac{-1}{12}.$$

19) a) Boundary is in two pieces $\sigma_1(\theta) = (\cos\theta, \sin\theta)$

$\sigma_2(\theta) = (a\cos\theta, -a\sin\theta)$.

$$\oint_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = \int_0^{2\pi} \sin^2\theta + \cos^2\theta\,d\theta + \int_0^{2\pi} \frac{-a^2\sin^2\theta}{a^2} - \frac{a^2\cos^2\theta}{a^2}\,d\theta$$

$= 0.$

dbl integral is $\iint_D \frac{\partial}{\partial x}\frac{x}{x^2+y^2} - \frac{\partial}{\partial y}\frac{-y}{x^2+y^2} = \iint_D dx\,dy = 0.$

b) On the other hand if we ignore σ_2 we get

$$\oint_C \frac{-y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy = 2\pi$$

while $\iint_D \frac{\partial}{\partial x}\frac{x}{x^2+y^2} - \frac{\partial}{\partial y}\frac{-y}{x^2+y^2}\,dx\,dy = 0$. These aren't equal since you can't apply Green's Theorem in b) since F isn't defined on this D.

6.4) (a) $\nabla \times F = (0,0,0)$

(b) $\oint_{\sigma} F \cdot ds = \int_0^{2\pi} -\sin\theta(-\sin\theta) + \cos\theta(\cos\theta)\, d\theta$

$= 2\pi$

(c) Since the line integral on a closed path $\neq 0$.
F is not conservative.

(d) The conditions aren't met since the domain of F is not simply connected.