

1. (10) (**Show all work**). Let $T(x, y, z) = x^3 + y^4 - xyz^2$. Determine whether T is increasing or decreasing at the point $(1, -2, 1)$ in the direction of the vector $\mathbf{u} = (1/\sqrt{3}, -1/\sqrt{3}, -1/\sqrt{3})$.
2. (15) (**Show all work**). The level surface $G(x, y, z) = (x - 2)^4 + (y - 2)^4 + (z - 1)^2 = 3$ and graph of $f(x, y) = 4 - x^2 - y^2$ are two surfaces which intersect at the point $(1, 1, 2)$. Determine whether or not they are tangent at that point, that is, whether they have the same tangent plane.
3. (15) (**Show all work**).
 - (a) Let $\mathbf{g}(x, y) = (x + y, x^2 - y^2, x^3y)$. Find the derivative matrix $D\mathbf{g}(1, 2)$.
 - (b) Suppose that $\mathbf{f} : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ has derivative matrix $D\mathbf{f} = \begin{pmatrix} 1 - vw & 1 - uw & 1 - uv \\ 2u & 2v & 1 \end{pmatrix}$.
With \mathbf{g} as above, find $D(\mathbf{f} \circ \mathbf{g})(1, 2)$.
4. (15) (**Show all work**). Consider the path $\mathbf{c}(t) = (\sin(5t), \sqrt{3}\sin(5t), 2\cos(5t))$.
 - (a) For which values of α, β, γ is $\mathbf{c}(t)$ a flowline for the vector field $\mathbf{F}(x, y, z) = (\alpha z, \beta z, \gamma x)$?
 - (b) Compute the arclength of $\mathbf{c}(t)$ for t from 1 to 5.
5. (15) (**Show all work**). Consider the vector field $\mathbf{F}(x, y, z) = (2x, 3y^2z, y^3 + \sin z)$.
 - (a) Show that the vector field \mathbf{F} is a gradient field by finding a function f with $\mathbf{F} = \nabla f$ and $f(0, 0, 0) = 4$.
 - (b) Compute the curl, $\nabla \times \mathbf{F}$.
 - (c) Compute the divergence, $\nabla \cdot \mathbf{F}$.
6. (15) (**Show all work**). Show that the path $\mathbf{c}(t) = (3e^t - 3, \sin t + 3, t^4/4 + t - 2)$ is tangent to the surface $F(x, y, z) = x^5 + y^2 - 3z^2 + xyz = -3$ at the point corresponding to $t = 0$. (In particular, this means that the tangent line to the curve would have to lie in the tangent plane to the surface).
7. (15) Parametrize the curve which is the intersection of the level surface $-17x^2 + 9y^2 + 2z^2 + 25 = 0$ and the plane $3x + z = 1$. *Hint*: the curve is an ellipse.