12. Compute $\oint_C y\,dx + xz\,dy + x^2\,dz$ where C is the boundary (oriented counterclockwise as viewed from the point $(0, 0, 8)$) of the portion of the plane $x+y+z=1$ in the first octant.

13. Verify Stoke's theorem for the vector field $\mathbf{F} = (2z, 3x, 5y)$ and the portion of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$, oriented by upward pointing normal vectors.

14. Let $\mathbf{F} = (xz - y, yz, x^2 + y^2)$. Compute $\oint_C \mathbf{F} \cdot d\mathbf{s}$ where C is the curve of intersection of the cylinder $x^2 + y^2 = 1$ with the graph of $z = x^2 - y^2$. Assume that C is oriented clockwise as viewed from the point $(0, 0, 10)$.

15. Let D be a region in $\mathbf{R}^3$ with boundary a closed surface S. Show that the flux of the vector field $\mathbf{F} = (x, y, z)$ outward through S is the volume of D times some constant. What is the value of the constant?

16. Find the outward flux of $\mathbf{F} = (x^2, y^2, z^2)$ across the boundary of the cylindrical shell $1 \leq x^2 + y^2 \leq 4$, $0 \leq z \leq 2$.

17. If fluid flow is given by the vector field $F(x, y, z) = (x, 0, 0)$ (in meters per second), how many cubic meters of fluid per second are crossing the upper half of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad z \geq 0,$$

with upward normal orientation. For the parametrization of an ellipsoid, see p. 428, number 8.

18. Let S be a surface with boundary curve C. Suppose the vector field F restricted to C is perpendicular to C. Explain why

$$\iint_S \text{curl } F \cdot \mathbf{n} = 0,$$

for either orientation of the surface S.