MIDTERM #1 QUESTIONS

(1) Let $f(x,y) = xy + ye^x$.

(a) Calculate ∇f.

(b) At the point $(0,2)$, in what direction does the function increase most rapidly? Give your answer as a unit vector.

(c) In the direction you found in (b), what is the magnitude of the rate of change of the function (ie, the value of the directional derivative in that direction)?

(d) Find an equation for the tangent plane to $z = f(x,y)$ at the point $(0,2,2)$.

(2) Let $R = [0,1] \times [0,2]$. Recall that $[[x]]$ is the largest integer less than or equal to x. Evaluate the double integral

$$\iint_R [[x+y]]\, dA.$$

(3) Evaluate the double integral

$$\int_0^1 \int_{2x}^2 e^{-y^2}\, dy\, dx.$$

(4) Consider the following equation, which expresses a double integral as a sum of iterated integrals:

$$\iint_D y\, dA = \int_{-1}^0 \int_0^{e^x} y\, dy\, dx + \int_0^{\pi/2} \int_0^{\cos x} y\, dy\, dx.$$

(a) Sketch the domain D.

(b) Evaluate the double integral. You may want to use the identity $\cos^2 x = (1+\cos 2x)/2$.

(5) Let D be the region bounded by $y = x, y = x\sqrt{3}$, and $x^2 + y^2 = 9$, located in the first quadrant. Evaluate

$$\iint_D \cos(x^2+y^2)\, dA.$$

(6) Let D be the upper half of the disc $x^2 + y^2 \leq 1$ (that is, the part of the disc with $y \geq 0$). Suppose the lamina Ω fills the region D, and has density given by $\rho(x,y) = |x|$.

(a) Calculate the mass of the lamina.

(b) Find the coordinates of the center of mass of the lamina.