MIDTERM #2 QUESTIONS

(1) (15) Let E be the solid bounded by $x \geq 0, y \geq 0, 0 \leq z \leq 1 - x^2 - y$.

    (a) Sketch the projection of E onto the xy, xz, and yz planes. You do not need to explain why your answer is correct, but label all intercepts and identify the curves on the boundary of the regions you sketch by writing down the functions they are given by.

    (b) Write the triple integral $\iiint_E x \, dV$ as an iterated integral in each of the orders of integration $dz \, dy \, dx, dy \, dz \, dx, dx \, dy \, dz$.

    (c) Evaluate the triple integral $\iiint_E x \, dV$ using any of the orders you found in part (b).

(2) (15) Consider the cone $0 \leq z \leq 2 - \sqrt{x^2 + y^2}$. Let E be the part of this cone with $0 \leq z \leq 1$ (that is, we cut away the top part of the cone).

    (a) Suppose we want to evaluate $\iiint_E f(x, y, z) \, dV$ using cylindrical coordinates. Write down a sum of two iterated integrals, one of which corresponds to the bounds $0 \leq r \leq 1$ on r, and the other with $1 \leq r \leq 2$, which is equal to this triple integral.

    (b) With $f(x, y, z) = 1$, evaluate $\iiint_E dV$ to find the volume of E.

    (c) The volume of a cone is given by the formula $V = \dfrac{bh}{3}$, where b is the area of the circular base and h the height of the cone with respect to this base. Use this formula from geometry (no integration allowed!) to compute the volume of E and check that your answer is the same as that in part (b).

(3) (10) A solid fills the hemisphere E given by $x^2 + y^2 + z^2 \leq 4, z \geq 0$, and has density function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$. Find the mass of the solid.

(4) (10) Suppose a change of variables T from the uv plane to the xy plane is given by the formulas

$$
\begin{aligned}
x &= u + 3v \\
y &= 2u + 5v
\end{aligned}
$$

    (a) Find the Jacobian of this change of variables.

    (b) Let S be the rectangle in the uv plane given by inequalities $0 \leq u \leq 2, 0 \leq v \leq 1$. You may assume that the image of a rectangle under the transformation T is a parallelogram. Find the vertices of this parallelogram.

    (c) What is the area of the parallelogram you found in (b)? Remember to explain how you found your answer.

(5) (10) Let C be the semicircle $x^2 + y^2 = 4, y \geq 0$. Evaluate the line integral

$$
\int_C xy + x + y \, ds.
$$

(6) (10) Let $\mathbf{F} = \langle y^2, x \rangle$.

(a) Let C be the line segment starting at $(0,0)$ and ending at $(4,2)$ – that is, the orientation of C is given by starting at $(0,0)$ and ending at $(4,2)$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(b) Let C now be given by the piece of the graph of $y = \sqrt{x}$ starting at $(0,0)$ and ending at $(4,2)$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(c) Based on your answers to parts a and b, is $\mathbf{F}$ conservative? Briefly explain why.

(7) (10) Let $\mathbf{F}(x,y) = \langle y \cos x^2 + y^2 + 2xy, x^2 + 2xy + 2 \rangle$. Let C be the closed curve given by the triangle, starting at $(0,0)$, then going to $(2,2)$, then $(2,0)$, and the back to $(0,0)$. Use Green's Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

(8) (10) For each of the following five vector fields and corresponding domains D they are defined on, determine whether they are conservative on D or not, and explain why.

(a) $\mathbf{F}(x,y) = \langle 3xy, x^2 \rangle, D = \mathbb{R}^2$.

(b) $\mathbf{F}(x,y,z) = \langle x, y, z \rangle, D = \mathbb{R}^3$.

(c) $\mathbf{F}(x,y) = \langle e^{x^2} + y, e^{y^2} \rangle$, D is the annulus $1 \le x^2 + y^2 \le 4$.

(d) $\mathbf{F}(x,y) = \langle \log xy + 1 + y^2 \cos x, \frac{x}{y} + 2y \sin x \rangle, D = \{(x,y) | x, y > 0\}$; ie, D is the first quadrant, not including the x, y axes.

(e) $\mathbf{F}(x,y) = \langle \dfrac{-y}{x^2 + y^2}, \dfrac{x}{x^2 + y^2} \rangle$, D is given by the inequalities $x > 0, -x < y < x$.

(9) (6) For each of the three shaded regions below, indicate whether they are $\mathbf{A}$: connected but not simply connected, $\mathbf{B}$: simply connected, or $\mathbf{C}$: neither connected nor simply connected. You do not need to explain your answer.

(a) (A bubble letter A)

(b) (A bubble letter S)

(c) (A bubble exclamation point)

(10) (4) Match each function with the plot of its gradient vector field. Write the letter attached to a function underneath its corresponding plot. You do not need to explain your answers.

$\mathbf{A} : f(x,y) = x + y$
$\mathbf{B} : f(x,y) = x^2 + y^2$
$\mathbf{C} : f(x,y) = xy$
$\mathbf{D} : f(x,y) = e^x$