Final Exam Topics
Math 22, Spring 2007

Sections covered: 1.1–1.5, 1.7–1.9, 2.1–2.3, 3.1, 3.2, 4.1–4.7, 5.1–5.4, 6.1–6.3, 6.5, 6.6

What have we talked about this term? Let me try to organize the concepts.

- Linear equations
  - Vocabulary: coefficient, system, solution (set), equivalence, (in)consistency, (non)homogeneous, trivial solution
  - Methods: allowed operations, conversion to a matrix-vector product, conversion to an augmented or coefficient matrix
  - Theory: possible numbers of solutions
- Vectors
  - Basic: equality, sum, scalar multiple, geometric interpretation, linear combinations, weights
  - Sets of vectors: linear (in)dependence, span, closure under sum and scalar multiple, representing sets of vectors as parametric vector equations
  - Main example: $\mathbb{R}^n$ and subspaces thereof
  - Other significant examples: $\mathbb{P}$, $\mathbb{P}_n$, $M_{m\times n}$
  - Showing subspace: image set or preimage set of a subspace under a linear transformation is a subspace; the span of a set of vectors is a subspace; showing closure and containment of $\mathbf{0}$ (or closure and nonemptiness) proves a subset is a subspace
  - Bases: dimension, coordinate vectors, change of basis, relationship between dimension, spanning, and linear independence
  - Dot product: length/norm, unit vector, normalization, distance
  - Orthogonality: orthogonal complement, orthogonal set/basis, orthonormal set/basis, orthogonal projection, projections as approximations
  - Least squares method: how to find a best-fit line for a set of points
- Matrices
  - Basics: size, notation for entries, main diagonal, equality, sum, scalar multiple, product with a vector or another matrix (and how matrix multiplication's properties differs from standard arithmetic multiplication)
  - Relation to linear equations: coefficient matrix, augmented matrix, row reduction, row equivalence, (reduced) echelon form, pivot position/column, leading entry/variable, free variable
  - Inverses: inverse of a product, inverses for 2×2 matrices, general procedure for finding inverses, connection to linear transformations, equivalent conditions to invertibility
  - Determinants: cofactor expansion, relation to invertibility, determinants for triangular matrices
  - Associated vector spaces: row space, column space, null space; rank, finding bases, relation to invertibility
- Linear Transformations
  - Vocabulary: domain, codomain, range, image, preimage, one-to-one, onto, linear

  - Matrix transformations: finding matrices for transformations, matrices for non-$\mathbb{R}^n$ transformations via coordinate vectors, connection between transformation properties and properties of matrix columns as a set of vectors
- Eigenstuff
  - Vocabulary: eigenvector, eigenvalue, eigenspace, characteristic function
  - Similarity: preservation of eigenvalues, diagonalizability, interpretation of diagonalizability with respect to bases and linear transformations
  - Multiplicity: relationship between algebraic (root of characteristic function) and geometric (dimension of eigenspace) and what that means for diagonalizability
- Things to skip
  - calculation of determinants by row reduction
  - all mentions of difference equations (chp 5 in particular has these at the ends of sections)
  - last segment of §6.1
  - last segment of §6.5
  - all but first segment of §6.6