Ungraded Quiz + Questionnaire 3

Your name: __________

April 25, 2014

1. Is $\{1, t\}$ a basis for the vector space $\mathbb{P}_2$ of all polynomials with degree ≤ 2?

   NO. Any basis for $\mathbb{P}_2$ has three elements, because the basis $\{1, t, t^2\}$ has three elements. Thus $\{1, t\}$ is not a basis for $\mathbb{P}_2$.

2. True or false: if B is a basis for V and $\mathbf{v} \in V$, then for any scalar $c \in \mathbb{R}$, $[c\mathbf{v}]_B = [\mathbf{v}]_B$.

   MOST EXCEEDINGLY FALSE. In general we have $[c\mathbf{v}]_B = c[\mathbf{v}]_B$; this corresponds to the fact that sending a vector in V to its B-coordinates is a linear transformation. For example, if $V = \mathbb{P}_2$ and $B = \{1, t, t^2\}$, then $[1 + 2t + 3t^2]_B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $[10(1 + 2t + 3t^2)]_B = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$.

3. Let A be a matrix with one million rows and 2017 columns. Is it possible that $\dim \operatorname{nul} A = 100$ and $\dim \operatorname{col} A = 200$?

   IMPOSSIBLE. If A has 2017 columns then

   $$\dim \operatorname{nul} A + \dim \operatorname{col} A = 2017.$$

   A notable fact about the numbers 200 and 200 is that their sum is not 2017. Thus we cannot have both $\dim \operatorname{nul} A = 100$ and $\dim \operatorname{col} A = 200$.