Math 22: Exam 2

October 30, 2012, 6pm-8pm

Your name (please print): ____

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. Unless otherwise stated, you must justify all of your answers to receive credit - please write in complete sentences in a paragraph structure. You may not give or receive any help on this exam and all questions should be directed to Professor Pauls.

You have **2 hours** to work on all **8** problems. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Problem	Points	Score
1	10	
2	15	
3	12	
4	10	
5	10	
6	15	
7	13	
8	15	
Total	**100**	

(1) (10 points, 5 each) Let A be an $n \times m$ matrix.

(a) Define the null space, the column space and the row space of A.

(b) Identify the vector space of which *Row A* is a subset. Show that *Row A* is a subspace of that vector space.

(2) (15 points) Which of the following matrices are invertible? For the 2×2 matrices, if they are invertible, find their inverses. In each case justify your answer!

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 1 \\ -2 & 1 & 0 \\ 4 & -2 & 7 \end{pmatrix}, \quad \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$$

(3) (20 points total) The matrix A given by

$$\begin{pmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{pmatrix}$$

has reduced echelon form given by

$$\begin{pmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(a) (2 points) Find a basis, $\mathfrak{B}$, for *Row* A.

(b) (3 points) If we label your basis from part a) as $\mathfrak{B} = \{\vec{b}_1, \ldots, \vec{b}_k\}$, is $\{A\vec{b}_1, \ldots, A\vec{b}_k\}$ a basis for $Col\ A$? Justify your answer!

(c) (3 points) Find a basis, $\mathfrak{N}$, for $Nul\ A$.

(d) (3 points) Show that the union of the vectors in $\mathfrak{N}$ and $\mathfrak{B}$ is a basis for $\mathbb{R}^4$.

(e) (1 points) Give a change of basis matrix from the standard basis to the basis in the previous part (if the answer is the inverse of a matrix, you need not compute the inverse).

(4) (10 points total) Consider the matrix A given by

$$\begin{pmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{pmatrix}$$

A has a double eigenvalue of 3.

(a) (5 points) Find all of the eigenvectors associated with the eigenvalue 3.

(b) (5 points) Is A diagonalizable? If not, why? If so, show the diagonalization.

(5) (10 points total) Let A be

$$\begin{pmatrix} 2 & -2 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{pmatrix}$$

and

$$\vec{b} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}.$$

Solve $A\vec{x} = \vec{b}$ using the LU factorization for A given by

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 4 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 2 \\ 0 & -3 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

(6) (15 points total) Let P be a regular Markov chain given by

$$\begin{pmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{pmatrix}$$

(a) (5 points) Find all the eigenvalues and associated eigenvectors of P.

(b) (10 points) What does $P^k\vec{x}$ converge to as k approaches infinity? Justify your answer completely!

(7) (13 points total) Consider the linear transformation $T : P_2 \to \mathbb{R}^3$ given by

$$T(a_0 + a_1 x + a_2 x^2) = (a_0 + a_1, 3a_1 + 2a_2, 4a_2)$$

(a) (4 points) Using the basis $\mathfrak{B} = \{1, x, x^2\}$ for P_2 and the standard basis $\mathscr{E}$ for $\mathbb{R}^3$, give a matrix representation, A, for T with respect to these two bases.

(b) (2 points) What is the rank of T? Justify your answer!

(c) (5 points) Find the eigenvalues and eigenvectors of A. If possible, give a diagonalization of A.

(d) (2 points) Is A invertible? Justify your answer.

(8) (15 points total)

(a) (5 points) Show that if A and B are similar matrices then $det\ A = det\ B$.

(b) (5 points) Show that similar matrices have the same eigenvalues.

(c) (5 points) Explain why an $n \times n$ matrix can have at most n distinct eigenvalues.

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