Math 23 Diff Eq: Homework 8

due Wed Nov 23 (or before you leave for break)
. . . but best if do relevant questions after each lecture

Note on integrals: these days it's professional to check your integrals symbolically, especially since you have a bunch to do to get Fourier coefficients. You could use Matlab's Symbolic Toolbox (separate licence but Dartmouth may have). Or here's example commands in (free) Maple to compute $\int_{-L}^{L} x \sin(n\pi x/L)\, dx$.

```
assume(n,integer);
f := x*sin(n*x*Pi/L);
A := int(f,x=-L..L);
```

Gives answer $2(-1)^{n+1}L^2/n\pi$. How great is that? Not required for our course–this is purely to help you out!

A: Let's finish off the "*Linearization... again!*" worksheet question I gave you on 11/11/05. Please find the critical points of

$$\begin{aligned} x' &= xy - y \\ y' &= x - x^2 - y \end{aligned}$$

and categorize the linearized behavior (A matrix) at each critical point. For the non-$\mathbf{0}$ critical point, does the linear system allow you to predict stability? [Hint: imaginary axis]. Check this by using `pplane7` or its applet to plot the (beautiful) trajectories. Is this point in fact stable?

9.3: 7.

10.2: 13 (sawtooth wave), 19 (see Example in 10.3. For the plot you can use the Fourier applet on square-wave setting; you don't need to write Matlab code).

B: Let's derive the orthogonality properties of sin and cos on p. 578. First evaluate $\int_{-1/2}^{1/2} e^{2\pi iky}dy$ for k integer (consider $k = 0$ too). Now write sin and cos using Euler's formula. Then expand $\int_{-1/2}^{1/2} \cos(2\pi ny)\cos(2\pi my)dy$ using Euler's formula, and treat the 4 terms using your first result. Finally, change variable $x = 2Ly$. Repeat for the other two orthogonality integrals. This will be painless.

10.3: 2 (Consider the Theorem when drawing the sketch. Watch out for the way series is written in back; you will find expressions such as $\cos n\pi = (-1)^n$ for integer n useful), 17.

10.4: 1, 6, 7, 27 (for c & d, you don't need to plot. Instead just answer d by comparing triangle vs sawtooth waves on the applet, or comparing Fig. 10.2.4 and Fig. 10.4.3).

10.5: 3, 7, 9 (your answer should be an infinite sum; it would be nice if you simplified $(1 - \cos n\pi)$).