MATH 23 Exam 2 Review Problems

Problem 1. *Use the method of reduction of order to find a second solution of the given differential equation*

$$x^2 y'' - (x - 0.1875)y = 0, \quad x > 0, \quad y_1(x) = x^{1/4} e^{2\sqrt{x}}$$

Problem 2. *Consider the initial value problem*

$$4y'' + 4y' + y = 0, \quad y(0) = 1, \quad y'(0) = 2.$$

(a) Solve the initial value problem and plot the solution.

(b) Determine the coordinates (t_M, y_M) of the maximum point.

(c) Change the second initial condition to $y'(0) = b > 0$ and find the solution as a function of b.

(d) Find the coordinates (t_M, y_M) of the maximum point in terms of b. Describe the dependence of t_M and y_M on b as b increases.

Problem 3. *Solve the given initial value problem. Sketch the graph of the solution and describe its behaviour for increasing t.*

$$9y'' + 6y' + 82y = 0, \quad y(0) = -1, \quad y'(0) = 2$$

Problem 4. *Find the general solution of the given differential equation*

$$u'' + \omega_0^2 u = \cos \omega t, \quad \omega^2 \neq \omega_0^2$$

Problem 5. *Determine a suitable form of particular solution $Y(t)$ using the method of undetermined coefficients*

$$y'' + 3y' + 2y = e^t (t^2 + 1) \sin 2t + 3e^{-t} \cos t + 4e^t$$

Problem 6. *Verify that the given functions y_1 and y_2 satisfy the corresponding homogenous equation; then find a particular solution of the given nonhomogeneous equation.*

$$x^2y'' - 3xy' + 4y = x^2 \ln x, \quad x > 0, \quad y_1(x) = x^2, \quad y_2(x) = x^2 \ln x$$

Problem 7. *Verify that the given functions y_1 and y_2 satisfy the corresponding homogenous equation; then find a particular solution of the given nonhomogeneous equation.*

$$x^2y''+xy'+(x^2-0.25)y = g(x), \quad x > 0; \quad y_1(x) = x^{-1/2} \sin x, \quad y_2(x) = x^{-1/2} \cos x$$

Problem 8. *A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in, and then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position u of the mass at any time t. Determine the frequency, period, amplitude, and phase of the motion.*

Problem 9. *A 1/4-kg mass is attached to a spring with a stiffness 4 N/m. The damping constant b for the system is 1 N-sec/m. If the mass is displaced 1/2 m to the left and an initial velocity of 1 m/sec to the left, find the equation of motion. What is the maximum displacement that the mass will attain?*

Problem 10. *Verify that the given vector satisfies the given differential equation*

$$x' = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} x, \quad x = \begin{bmatrix} 6 \\ -8 \\ -4 \end{bmatrix} e^{-t} + 2 \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} e^{-2t}$$

Problem 11. *Verify that the given matrix satisfies the given differential equation*

$$\psi' = \begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & 1 \end{bmatrix} \psi, \quad \psi(t) = \begin{bmatrix} e^t & e^{-2t} & e^{3t} \\ -4e^t & -e^{-2t} & 2e^{3t} \\ -e^t & -e^{-2t} & e^{3t} \end{bmatrix}$$

Problem 12. *Verify that the given functions are solutions of the differential equation, and determine the Wronskian*

$$xy''' - y'' = 0; \quad 1, \quad x, \quad x^3$$

Problem 13. *Determine intervals in which solutions are sure to exist*

$$(x-1)y^{(4)} + (x+1)y'' + (\tan x)y = 0$$

Problem 14. *Find the general solution of the given differential equation*

$$y^{(6)} - y'' = 0$$

Problem 15. *Find the general solution of the given differential equation*

$$y^{(8)} + 8y^{(4)} + 16y = 0$$

Problem 16. *Determine a suitable form for the particular solution ($Y(t)$) if the method of undetermined coefficients is to be used. Do not evaluate the constants.*

$$y^{(4)} + 2y''' + 2y'' = 3e^t + 2te^{-t} + e^{-t}\sin t$$

Problem 17. *Find the solution of the initial value problem*

$$y''' - 3y'' + 2y' = t + e^t; \quad y(0) = 1, \quad y'(0) = -1/4, \quad y''(0) = -3/2$$

Problem 18. *Given that x, x^2, and $1/x$ are solutions of the homogeneous equation corresponding to*

$$x^3y''' + x^2y'' - 2xy' + 2y = 2x^4, \quad x > 0,$$

determine a particular solution.

Problem 19. *Find the solution of the given initial value problem*

$$y''' + y' = \sec t; \quad y(0) = 2, \quad y'(0) = 1, \quad y''(0) = -2$$

Problem 20. *Find all eigenvalues and eigenvectors of the given matrix*

$$\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$$

Problem 21. *Find all eigenvalues and eigenvectors of the given matrix*

$$\begin{bmatrix} 11/9 & -2/9 & 8/9 \\ -2/9 & 2/9 & 10/9 \\ 8/9 & 10/9 & 5/9 \end{bmatrix}$$