

1. (15) (**Show all work**).

Find the general solution to the following differential equation:

$$y' + 3y = t + e^{-2t}$$

2. (15) (**Show all work**). Find the solution to the differential equation

$$y'' - 4y' + 4y = 0$$

Subject to the initial conditions:

$$y(0) = 2, \quad y'(0) = -1$$

3. (20) (**Show all work**). Find the Fourier series representation for

$$f(x) = \begin{cases} x & -\pi \leq x < 0 \\ 0 & 0 \leq x < \pi \end{cases}$$

and so that $f(x + 2\pi) = f(x)$.

4. (20) (**Show all work**).

Solve the system of equations

$$x_1'(t) = 5x_1(t) - x_2(t)$$

$$x_2'(t) = 3x_1(t) + x_2(t)$$

Subject to the initial conditions

$$x_1(0) = 2, \quad x_2(0) = -1$$

5. (30) (**Show all work**).

Solve the following partial differential equation:

$$a^2 u_{xx} - \alpha^2 u = u_{tt}$$

for a function $u(x, t)$ where a and α are real constants and u satisfies the following boundary conditions:

$$u(0, t) = 0$$

$$u(1, t) = 0$$

$$u_t(x, 0) = 0$$

$$u(x, 0) = f(x)$$