Miscellaneous thoughts on the midterm.

$\star$ Remember with the true-false that every statement is in its full generality. If a statement holds for some but not all instances, it is false.

$\star$ Remember the difference between a coordinate vector and the vector it represents – in the standard basis the two often look very much alike but they are not the same, and with a nonstandard basis they will generally be quite different.

$\star$ To prove two sets are equal it is often useful to prove in two steps; that is, containment each direction.

$\star$ A different way to look at matrix multiplication which may be helpful:

Each row of AB is a linear combination of the rows of B with coefficients from the corresponding row of A:
If the second row of A is 02000, the second row of AB will be twice the second row of B.
If the fourth row of A is 10001, the fourth row of AB will be the sum of the first and fifth rows of B.

Likewise, each column of AB is a linear combination of columns of A with coefficients from the corresponding column of B:
If the first column of B is 01010, the first column of AB will be the sum of the second and fourth columns of A.
If the third column of B is 20002, the third column of AB will be twice the sum of the first and fifth columns of A.

Those examples could all have come from the following matrix multiplication:

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 2 & 4 \end{pmatrix}.$$

This gives also a different way to look at the identity matrix: since the first row and column of I_n are both $10\ldots0$, the first row of I_nB will be equal to the first row of B and the first column of AI_n will be equal to the first column of A. Likewise as you move from 1 to n, for both rows and columns, and hence multiplication on either side will not change A or B.

⋆ Big hint for 2.3 #17:
Projections are the answer (see the bottom of p. 76). Note that T^2 here means T composed with T. Verify it by (the easy direction) showing every projection is equal to its composition with itself, and (the harder direction) everything which is equal to its composition with itself is a projection. For the latter you can use the hint in the book, saying that $x = x - T(x) + T(x)$; by assumption on T, this is the sum of an element of N(T) and an element of $\{y : T(y) = y\}$. You need to show that the latter is a subspace, and that it intersects $N(T)$ only at *0*. Then V is the direct sum, and T is just the projection along $N(T)$ onto $\{y : T(y) = y\}$.

⋆ For 2.2 #11, the matrix shown there is simply meant to convey the idea that the first k entries of the bottom $n - k$ rows are all zero. For #15, remember functions (and linear transformations, therefore) are defined entirely by what they do to their inputs. $aT + U$ is the function which takes any x to $aT(x) + U(x)$, by definition, and that is all you need to know about it.