

Notes on Proofs

Math 24 Spring 2006

Now that I've seen what some of the common difficulties or mistakes are, I can address them more directly.

Careful! Don't insert uniqueness where it's not asked for. In general if you are not asked to prove something is unique that means under the hypotheses given it may not be.

Audience: Please write the statement to be proved; the reader is assumed to be familiar with the relevant definitions and basic results, but not reading your mind as to what statement you are proving. As for that first assumption, please do make it, and don't restate the definitions at length.

Boldness: There is a place for words like *would*, *could*, *should*, *might*, and *ought* in proofs, but they should be kept to a minimum. Most of the time the proof is clearer if you stick to *has*, *will*, *does*, and *is*. This is especially true in proofs by contradiction. Since in such a proof you are assuming something which is not true, it may feel more natural to use the subjunctive, but it can come across as tentative. You assume some hypothesis; given that hypothesis other statements *are* or *are not* true. Be bold and let the whole contraption go up in flames when it runs into the statement it contradicts.

Sentence construction: If a sentence seems strained, try turning it around. That is, try putting descriptive phrases before the noun they modify instead of after (or vice-versa), and putting clauses of the sentence into a different order. Rearranging several consecutive sentences may also be helpful. Do not fear to edit: the goal is a readable proof that doesn't require too much back-and-forth to understand.

One small thing: Unless otherwise stated, in the proof of a theorem the phrase "the theorem" refers to the theorem being proved. No need to use explanatory phrases (likewise for propositions, lemmas, corollaries, etc.).

Notes revised from the syllabus: It is frequently awkward to use symbols as abbreviations for the phrases often used in their stead when reading mathematics aloud. However, it can be clumsy and expand proofs out of readability to avoid symbols altogether. Striking this balance is one of our goals.

If you have a definition before you of a particular concept and are asked to prove something about the concept, you must stick to the definition. Finally, be wary of mentally adding not only uniqueness, but phrases like *only*, *for all*, *for every*, or *for some* which are not actually there.