Dear Mr. Stemko,

Thank you for contacting me regarding your problem. As you no doubt recall, you want to know the area of a racetrack to determine how much asphalt you will need. The area you will need to pave is approximately 34,451 square feet. I am assuming that the racetrack is in fact an exact rectangle with exact half-circles attached to the sides, while the actual track may be a little bit less than perfect.

To find the area that you will need to pave, I first determined the area of the entire region, then subtracted the area of the interior of the racetrack (which you will not be paving), which leaves exactly the part you are interested in (shaded gray on the diagram below).
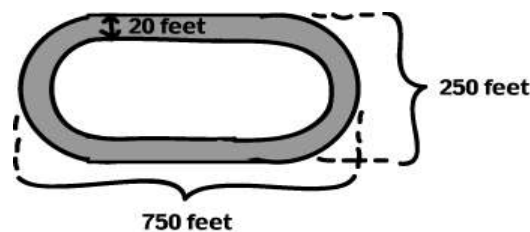
Diagram of The Racetrack

There are only two formulas we will need to use: the area of a circle is approximately 3.1416 times its radius squared, and the area of a rectangle is its height times its width. Either of these formulas can be found in any geometry textbook.

To find the area of the entire region, we first divide it up into a rectangle and two half-circles. Since the height of the racetrack is also the diameter of the half-circles, we know that the radius of each half-circle is (1/2)(250 ft.) = 125 ft. The height of the racetrack is also the height of the rectangle. The width of the racetrack is the same as the radius of each half-circle plus the width of the rectangle (see the diagram below). Therefore, the width of the rectangle is (750 ft.) – (2)(125 ft.) = 500 ft.
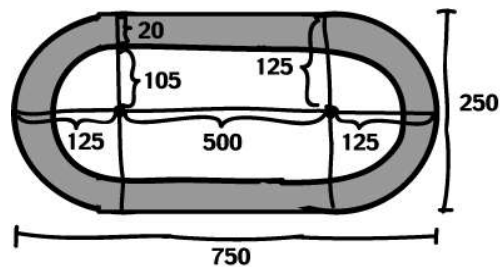
Diagram of the Racetrack
(all measurements are in feet)

Using the area formulas mentioned above, I calculated that the area of the total region is about

$$(3.1416)(125 \text{ ft.})^2 + (250 \text{ ft.})(500 \text{ ft.}) = 174{,}087.5 \text{ square feet.}$$

Finding the area of the interior of the racetrack is calculated in a similar way. The radius of the interior half-circles is (125 ft.) - (20 ft.) = 105 ft. The width of the interior rectangle is still 500 ft., but the height is (250 ft.) - (2)(20 ft.) = 210 ft. Therefore, the area of the interior region is

$$(3.1416)(105 \text{ ft.})^2 + (210 \text{ ft.})(500 \text{ ft.}) = 139{,}636.14 \text{ square feet.}$$

Hence, the area which you need to pave is 174,087.5 – 139,636.14 = 34,451.36 sq ft.

I hope my calculations were of assistance.

Regards,

Mathematics Student

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