First Exam

MATH 2 — WINTER 2014

NAME:

SECTION: **11** **2**

This exam has 8 questions on 9 pages, for a total of 30 points.

You have 120 minutes to answer all questions.

This is a closed book exam.

Use of calculators and other electronic devices is not permitted.

Show all your work, justify all your answers.

Question	Points	Score
1	30	
2	0	
3	0	
4	0	
5	0	
6	0	
7	0	
8	0	
Total:	30	

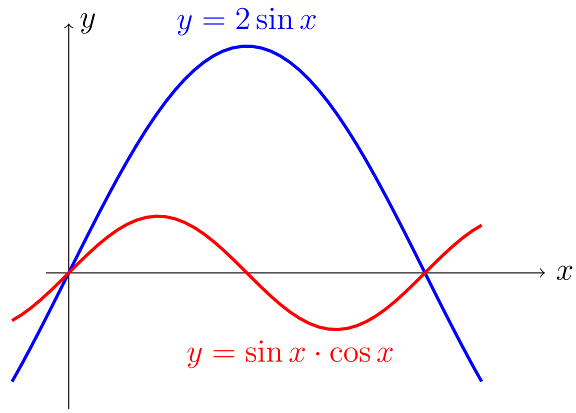
30 1. Compute the area of the region in the first quadrant bounded by the functions

$$y = 1 + x^2, \qquad y = 2, \qquad \text{and} \qquad x = 0.$$

☐ 2. Compute the volume of the region from Problem 1 rotated about the x-axis. Recall that the region is bounded in the first quadrant by the functions

$$y = 1 + x^2, \qquad y = 2, \qquad \text{and} \qquad x = 0.$$

□ 3. Compute the area between (or bounded by) the given curves.



☐ 4. Verify that the stated indefinite integrals are valid.

(a) $$\int xe^{-3x}\,\mathrm{d}x = -\frac{e^{-3x}}{3}\left(x+\frac{1}{3}\right)+C.$$

(b) $$\int e^{x}\cos x\,\mathrm{d}x = \tfrac{1}{2}e^{x}(\sin x+\cos x)+C.$$

☐ 5. Evaluate the following indefinite integrals.

(a) $\displaystyle\int \sin x \cos x \, \mathrm{d}x$

(b) $\displaystyle\int \frac{x^4 + 9}{3x^2} \, \mathrm{d}x$

(c) $\displaystyle\int \frac{3x^3 + 2x^2 e^x}{x^3 e^x} \, \mathrm{d}x$

☐ 6. Let $A(x) = \int_{2}^{x} e^{t^3}\, dt$. Calculate:

(a) $A(2)$

(b) $A'(x)$

☐ 7. Calculate:

(a) $\displaystyle\int_{-1}^{1} \frac{1}{1+x^2}\,dx$. *Hint: it may help to recall that* $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$.

(b) $\displaystyle\int_{-\pi/3}^{\pi/3} \sin(x) - x^3\,dx$

☐ 8. Let $f(x) = \sqrt{x}$ for $0 \leq x \leq 4$.

(a) Sketch the area that represents the 2-rectangle left-endpoint approximation to $\int_0^4 f(x)\,dx$.

(b) Sketch the area that represents the 2-rectangle right-endpoint approximation to $\int_0^4 f(x)\,dx$.

(c) Fill in the gaps in the following statements:

- The __________-endpoint approximation is an overestimate because $f(x)$ is _________________________.
- The __________-endpoint approximation is an underestimate because $f(x)$ is _________________________.

(d) We can form a new approximation to the area called the trapezoidal approximation by joining the left- and right-endpoints of each subinterval.

Sketch this for $\int_0^4 f(x)\,dx$ and fill in the blanks in the following:

The __________-endpoint approximation is an _______________estimate because $f(x)$ is _________________________.