Second Exam

MATH 2 — WINTER 2014

NAME:

SECTION: **11** **2**

This exam has 7 questions on 10 pages, for a total of 175 points.

You have 120 minutes to answer all questions.

This is a closed book exam.

Use of calculators and other electronic devices is not permitted.

Show all your work, justify all your answers.

There are some formulas you might find useful on the last page.

Question	Points	Score
1	15	
2	30	
3	25	
4	30	
5	30	
6	25	
7	20	
Total:	175	

15 1. Suppose that $\int_{-1}^{3} f(x)\,dx = 10$. What is the average value of $f(x)$ on the interval $[-1, 3]$?

30 2. Use integration by parts to evaluate:

(a) $\int x \ln(5x)\, dx$

(b) $\int \arctan(x)\, dx$

25 3. (a) Show that

$$\int \frac{1}{(4+x^2)^{3/2}}\,dx = \frac{1}{4}\sin(\arctan(x/2)) + C$$

(b) Simplify this answer using geometry.

30 4. Perform a trigonometric substitution for the following integrals, simplify the resulting integrals as much as you can, but **do not solve!**

(a) $\int x^2\sqrt{1-x^2}\,dx$

(b) $\int \frac{1}{\sqrt{1+4x^2}}\,dx$

(c) $\int \frac{x^2}{\sqrt{x^2-6}}\,dx$

30 5. Solve the following trigonometric integrals, and simplify your solution as much as possible; in other words, **express your solution as a number!**

(a) $\displaystyle\int_{\pi/2}^{6\pi} \sin^3(x)\cos^2(x)\,dx$

(b) $\displaystyle\int_{0}^{\pi/4} \sec^4(x)\tan^4(x)\,dx$

25 6. Use cylindrical shells to calculate the volume of the solid obtained by rotating the region in the first quadrant under the curve $y = e^{-x}$ for $0 \leq x \leq 1$ about the y-axis.

20 7. Integrate the following using any technique(s) you know:

(a) $\int \ln(x)^2\,dx$

(b) $\int \cos(\sqrt{x})\,dx$

Hint: try the inverse substitution $x = u^2$.

Scratch work

$$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$$

$$\sin^2(x)+\cos^2(x) = 1$$

$$\tan^2(x)+1 = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$2\sin(x)\cos(x) = \sin(2x)$$

$$\cos^2(x)-\sin^2(x) = \cos(2x)$$

$$\int \sin^n(x) = -\frac{1}{n}\sin^{n-1}(x)\cos(x)+\frac{n-1}{n}\int \sin^{n-2}(x)\,dx$$

$$\int \cos^n(x) = \frac{1}{n}\cos^{n-1}(x)\sin(x)+\frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$

$$\int \tan^n(x) = \frac{1}{n-1}\tan^{n-1}(x)-\int \tan^{n-2}(x)\,dx$$

$$\int \sec^n(x) = \frac{1}{n-1}\tan(x)\sec^{n-2}(x)+\frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx$$