Final Exam

MATH 2 — WINTER 2014

NAME:

SECTION: **11** **2**

This exam has 11 questions on 16 pages, for a total of 250 points.

You have 180 minutes to answer all questions.

This is a closed book exam. Use of calculators and other electronic devices is not permitted.

Show all your work, justify all your answers.

There are some formulas you might find useful on the last page.

Question	Points	Score
1	20	
2	20	
3	30	
4	20	
5	15	
6	30	
7	20	
8	30	
9	30	
10	25	
11	10	
Total:	250	

$\boxed{20}$ 1. Compute the following antiderivative: $\displaystyle\int x\cos(3x)\,dx$.

$\boxed{20}$ 2. Solve the following indefinite integral: $\displaystyle\int \frac{1}{x^2\sqrt{x^2-36}}\,dx$.

$\boxed{30}$ 3. (a) Evaluate the following integral: $\displaystyle\int \frac{-2x+24}{x^2+x-6}\,dx.$

(b) Setup the partial fraction decomposition for the following rational function, but **do not solve for the coefficients!**

$$\frac{2x^3-5x^2+3x-1}{x^4-16}$$

$\boxed{20}$ 4. **Carefully** determine whether the integral converges or diverges; if convergent, compute its value:

$$\int_0^3 \frac{1}{\sqrt{3-x}}\, dx$$

$\boxed{15}$ 5. Exactly one of these functions is a probability distribution function. Identify which and briefly explain why.

$$p(x) = \begin{cases} \frac{\sin(x)}{2}, & \text{if } 0 \leq x \leq 3\pi \\ 0, & \text{if } x < 0 \text{ or } x > 3\pi \end{cases}$$

$$q(x) = \begin{cases} x, & \text{if } 0 \leq x \leq 2 \\ 0, & \text{if } x < 0 \text{ or } x > 2 \end{cases}$$

$$r(x) = \begin{cases} \frac{1}{2\sqrt{x}}, & \text{if } 0 \leq x \leq 1 \\ 0, & \text{if } x < 0 \text{ or } x > 1 \end{cases}$$

$\boxed{30}$ 6. For each of the following volumes, sketch the area in the xy-plane and write down a definite integral that computes the volume obtained by rotating the area about the specified axis. *Do not calculate the integral.*

(a) The area enclosed in the first quadrant by the coordinate axes and the parabola $y = x^2 - 6x + 5$, rotated about the x-axis.

(b) The area enclosed by the lines $y = x$, $x = 3y$ and $y = 6$, rotated about the x-axis.

(c) The area enclosed between the curves $y = 2^x$ and $y = x + 1$, rotated about the y-axis.

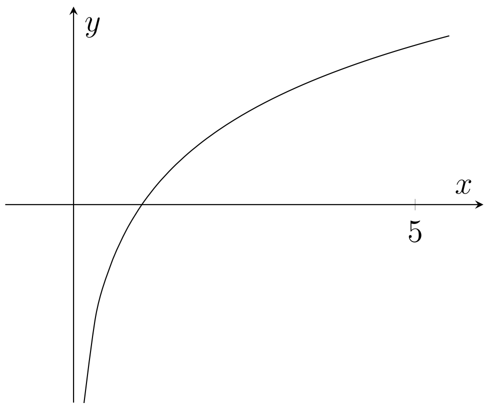
$\boxed{20}$ 7. Calculate the length of the curve $y = \frac{4}{3}x^{3/2}$ for $2 \leq x \leq 6$.

$\boxed{30}$ 8. Fill in the blanks in the following paragraph using some or all of the words listed below. You may use the same word multiple times, and more than one answer may be valid.

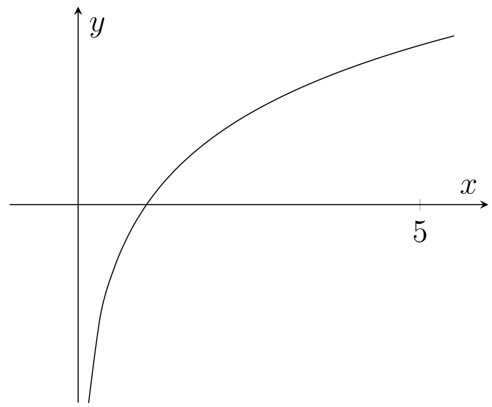
The area problem is expressed mathematically by the ______________ integral. The Fundamental Theorem of Calculus allows us to convert the problem of finding the ______________ area defined by a function into the easier problem of finding the ______________ of the function. In particular, the area (or area-so-far) function is a(n) ______________ of the original function.

definite	antiderivative	integration
signed	indefinite	absolute
integral	derivative	net

$\boxed{30}$ 9. (a) Sketch the areas on the graphs below that represent the left- and right-endpoint Riemann sum approximations to $f(x) = \ln(x)$ on $[1, 5]$ with 2 rectangles.



Left-endpoint approximation



Right-endpoint approximation

(b) Without doing any calculations, use your graphs to determine whether the left-endpoint or right-endpoint approximation is more accurate in this case.

(c) Give an example where the *other* approximation would be the more accurate one.

(d) What property of a function $f(x)$ determines which approximation is more accurate?

$\boxed{25}$ 10. Calculate:

(a) $$\int_{-5}^{5} 4 + \sqrt{25 - x^2}\, dx$$

(b) $$\int \frac{3}{x^{1/3}} - (\pi x)^2 + x^{-1}\, dx$$

$\boxed{10}$ 11. Calculate

$$\int e^{\sqrt{x}}\,dx$$

Scratch work

Scratch work

$$
\begin{aligned}
\frac{d}{dx}(\arcsin(x)) &= \frac{1}{\sqrt{1-x^2}} \\
\frac{d}{dx}(\arctan(x)) &= \frac{1}{1+x^2} \\
\sin^2(x)+\cos^2(x) &= 1 \\
\tan^2(x)+1 &= \sec^2(x) \\
\sin(x+y) &= \sin(x)\cos(y)+\cos(x)\sin(y) \\
\cos(x+y) &= \cos(x)\cos(y)-\sin(x)\sin(y) \\
\sin^2(x) &= \frac{1-\cos(2x)}{2} \\
\cos^2(x) &= \frac{1+\cos(2x)}{2} \\
2\sin(x)\cos(x) &= \sin(2x) \\
\cos^2(x)-\sin^2(x) &= \cos(2x) \\
\int \sin^n(x)\,dx &= -\frac{1}{n}\sin^{n-1}(x)\cos(x)+\frac{n-1}{n}\int \sin^{n-2}(x)\,dx \\
\int \cos^n(x)\,dx &= \frac{1}{n}\cos^{n-1}(x)\sin(x)+\frac{n-1}{n}\int \cos^{n-2}(x)\,dx \\
\int \tan^n(x)\,dx &= \frac{1}{n-1}\tan^{n-1}(x)-\int \tan^{n-2}(x)\,dx \\
\int \sec^n(x)\,dx &= \frac{1}{n-1}\tan(x)\sec^{n-2}(x)+\frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx \\
\int \tan(x)\,dx &= \ln(\sec(x))+C \\
\int \sec(x)\,dx &= \ln(\sec(x)+\tan(x))+C
\end{aligned}
$$